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## Rapid Solution of Evolution Equations in High Dimensions

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We analyze a discontinuous Galerkin time-stepping procedure for the numerical solution of nonlinear evolution equations in Hilbert or (reflexive) Banach spaces. Time steps and orders can be varied. For solutions which exhibit piecewise Gevrey regularity of index  $\delta$  in time (expressed by suitable countably normed spaces) we show that exponential convergence of order  $O(\exp(-bN^{1/(\delta+1)}))$  can be achieved, where  $N$  denotes the number of spatial problems to be solved.

Applications include the evolution of ‘Perfect incompressible fluids’ where  $\delta = 3$  and abstract parabolic equations where  $\delta = 1$ .

In the pricing of options on baskets of  $d$  underlyings or on indices, the spatial domains of the parabolic evolution problems arising in the Black-Scholes setting are often hypercubes in  $R^d$  with  $d$ , the number of underlyings, large (typically  $5 \leq d \leq 50$ ). We show how the DG time-stepping scheme can be combined with a sparse tensor product wavelet discretization in  $d$  dimensions to give an overall scheme with convergence rate  $O(h^p |\log h|^{(d-1)/2})$  with work of order  $O(h^{-1} |\log h|^{d+2})$ . Numerical experiments in dimensions  $d \leq 20$  on a personal computer confirm the theoretical results.

This is joint work with RISKLAB of ETH Zürich and with T. von Petersdorff of University of Maryland, College Park, USA.