Mathematical Modeling and Numerical Simulation of the Cardiovascular System

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The interest in the use of mathematical modeling and numerical simulation in the study of the cardiovascular system (and its inherent pathologies) has greatly increased in the past few years. Blood flow interacts mechanically and chemically with vessel walls producing a complex fluid-structure interaction problem, which is practically impossible to simulate in its entirety.

Several reduced models have been developed which may give a reasonable approximation of averaged quantities, such as mean flow rate and pressure, in different sections of the cardiovascular system. They are, however, unable to provide the details often needed for understanding a local behavior, such as, e.g., the effect on the shear stress distribution due a modification in the blood flow consequent to a partial stenosis.

In this talk we address some mathematical issues arising from the modeling of the cardiovascular system through problems of different complexity. The most complex model is based on the coupling of the Navier-Stokes equations with structural models for the vessel walls. An intermediate model is derived from integrating these equations on a section of a vessel geometry, and it is formed by a system of hyperbolic equations for the evolution of mean pressure and flow rate. An even simpler model that will be considered is based on the solution of a system of ordinary differential equations which describe electrical networks.

The derivation of these models will be presented together with schemes for their numerical solution. Furthermore, we will specifically address the coupling problem, analyzing different possible strategies. These techniques may be extended by including models for chemical transport. Some results in this direction are already available and will be presented.

The previous multi-scale approach looks a viable solution to obtain detailed numerical simulation of sections of the cardiovascular apparatus, while properly accounting for the presence of the global system. We expect that this technique will open new possibilities for the use of numerical modeling for medical research.