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## On the Recovery of a Surface with Prescribed First and Second Fundamental Forms

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The fundamental theorem of surface theory asserts that, if a field of positive definite symmetric matrices of order two and a field of symmetric matrices of order two together satisfy the Gauss and Codazzi-Mainardi equations in a connected and simply connected open subset of  $\mathbb{R}^2$ , then there exists a surface in  $\mathbb{R}^3$  with these fields as its first and second fundamental forms (global existence theorem) and this surface is unique up to isometries in  $\mathbb{R}^3$  (rigidity theorem).

The aim of this lecture, which is based on a recent joint work with François Larssonneur, is to provide a self-contained and essentially elementary proof of this theorem by showing how it can be established as a simple corollary of another well-known theorem of differential geometry, which asserts that, if the Riemann-Christoffel tensor associated with a field of positive definite symmetric matrices of order three vanishes in a connected and simply connected open subset of  $\mathbb{R}^3$ , then this field is the metric tensor field of an open set that can be isometrically imbedded in  $\mathbb{R}^3$  (global existence theorem) and this open set is unique up to isometries in  $\mathbb{R}^3$  (rigidity theorem).