On the Global Existence of Solution of Prandtl's System

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In this paper we establish a global existence of weak solutions to the 2-dimensional Prandtl's system for unsteady boundary layers in the class considered by O. A. Oleinik provided that the pressure is favorable is satisfied. The main results of Oleinik and her co-workers can be summarized as that there exists a unique (for short-time if L is given and fixed, and for arbitrary time if L is small) classical smooth solution to the initial-boundary value problem provided that the initial data satisfy conditions

$$U(x,t) > 0, \quad u_0(x,t) > 0, \quad u_1(y,t) > 0;$$

and

$$\partial_y u_0(x,y) > 0, \quad \partial_y u_1(x,y) > 0.$$

One of the open problems listed at the end of book by Oleinik and Samokhin is: what are the conditions ensuring the global in times existence and uniqueness of solution for arbitrary given L?

The main purpose of this paper is to establish the global (in time) existence of weak solution to the problem for arbitrary finite L and data satisfying

$$U(x,t) > 0, \quad u_0(x,t) > 0,$$

$$u_1(y,t) > 0, \quad v_0(x,t) \leq 0;$$

and

$$\partial_y u_0(x,y) > 0, \quad \partial_y u_1(x,y) > 0.$$

and provided that the pressure is favorable, that is,

$$\partial_x p(t,x) \leq 0.$$

This generalizes the local well-posedness results due to Oleinik. For the proof, we introduce a viscous splitting method so that the asymptotic behavior near the fluid can be estimated more accurately by methods applicable to the degenerate parabolic equations. This is a jointed work with Z.P. Xin.