
Uniqueness of BV Entropy Solutions for Quasilinear Parabolic Equations with Arbitrary Degeneracy

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In this paper we are concerned with the Cauchy problem for quasilinear parabolic equations of the form

$$\frac{\partial u}{\partial t} = \Delta A(u) + \nabla \vec{B}(u), \quad (x, t) \in Q_T, \quad (1)$$

$$u(x, 0) = u_0(x), \quad x \in \mathbb{R}^N, \quad (2)$$

where $Q_T \equiv \mathbb{R}^N \times (0, T)$, $A(s), \vec{B}(s)$ appropriately smooth. The basic assumption is that $A'(s) \geq 0$ which implies that the equation (1) may be arbitrarily degenerate. In other words, the equation is of parabolic and hyperbolic mixed type.

We are much interested in the uniqueness of BV entropy solutions, namely, the solutions with bounded variation satisfying some entropy conditions. First, we use a quite different new approach to define solutions. Roughly speaking, we choose the test functions in the same space, i.e., the space of all functions with bounded variation, as the solutions belong to, and incorporate with the entropy conditions into an integral inequality with an arbitrary test function and an arbitrary test constant. Using this new approach, we are able to separate the reasonable entropy conditions for the solutions. Then, based on the delicate properties of BV functions and BV_x functions, we establish the uniqueness of such solutions.