Asymptotic Behaviour of a Bingham Fluid in Thin Layers

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A Bingham fluid is a visco-plastic medium obeying the general laws of continuum mechanics and has a special nonlinear constitutive law. It is a non-Newtonian fluid which moves like a rigid body when a certain function of the stress tensor is below a certain threshold (sometimes called the yield stress). Beyond this yield stress, the fluid obeys a nonlinear constitutive law. In this paper, a nonlinear stationary model, in the form of a variational inequality giving the velocity and the pressure, is considered in a thin layer represented by the open set $(0,1) \times (0,\varepsilon)$ in the plane, where ε is a small parameter tending to zero. The original problem is then transformed into one posed over a fixed reference domain $(0,1) \times (0,1)$ thus bringing out the dependence on ε explicitly. The limit problem satisfied by the limits of the transformed variables as $\varepsilon \to 0$ is then studied. To do this we need to study the properties of a function space of Sobolev type. It is shown that the limit problem is well-posed in this function space. Finally, the differential equation satisfied by the limit variables in the 'non-rigid zone' is obtained and is compared with a one-dimensional model proposed in the engineering literature.