Fredholm Type Results for Nonlinear Operators

George Dinca
Faculty of Mathematics
University of Bucharest, Romania
E-mail: dinca@cnfis.ro

Consider an operator of the form $(\lambda T - S)$ where T and S are (generally nonlinear) operators from a Banach space X into Banach space Y, and T is invertible. The problem is then, to characterize those $\lambda \neq 0$ for which $(\lambda T - S)$ is surjective. To do so, two different methods are used.

The first one reduces the problem of the existence of a solution for the equation $(\lambda T - S)x = f$, to that of the surjectivity of the operator $(I - F)$, where

$$F \,:\, Y \longrightarrow Y, \quad Fy = S\big[T^{-1}\big(\frac{1}{\lambda}y\big)\big].$$

The arguments come from the spectral theory of nonlinear operators.

The second method reduces the same problem to that of the existence of a fixed point for the operator

$$A \,:\, X \longrightarrow X, \quad Ax = T^{-1}\big[\frac{1}{\lambda}(Sx + f)\big],$$

and the arguments come from the Leray-Schauder degree theory.

Some applications concerning the surjectivity for operators of the form $(\lambda J_\phi - S)$, where J_ϕ is a duality mapping, are considered. In particular, surjectivity results for $\lambda(-\Delta_p) - N_f$, where

$$\Delta_p u = \text{div } \big(|\, \nabla u\,|^{p-2} \nabla u\big)$$

is the so-called p-Laplacian, and

$$(N_f u)(x) = f(x, u(x)))$$

is the Nemytskii operator, are obtained.