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## Implicit PDEs and Existence of Minimizers in the Calculus of Variations

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We discuss the existence of Lipschitz solutions of implicit equations of the type

$$\begin{cases} F(x, u(x), Du(x)) = 0, \text{ a.e. in } \Omega \\ u = \varphi, \text{ on } \partial\Omega \end{cases}$$

where  $\Omega \subset \mathbb{R}^n$  is an open set,  $u : \Omega \rightarrow \mathbb{R}^m$  is vector valued,  $F : \Omega \times \mathbb{R}^m \times \mathbb{R}^{m \times n} \rightarrow \mathbb{R}$  is continuous and  $\varphi$  is a given Lipschitz map.

We then show how this analysis applies to minimization problems of the form

$$\inf \left\{ \int_{\Omega} f(x, u(x), Du(x)) dx : u = \varphi \text{ on } \partial\Omega \right\}.$$