On the Stationary Solution of the Mathematical Model for Grain Boundary Grooving

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In this talk, we will present some stationary solution for nonlinear partial differential equation called Mullins Equation which occurs in the theory of grain boundary grooving:

$$u_t = -C_1^E(u)(1+u_x^2)^{1/2} exp(-C_2^E(u)\frac{u_{xx}}{(1+u_x^2)^{3/2}}) + C_1^C(u)(1+u_x^2)^{1/2}.$$

The main tool, which we can use, is the admissibility property between weighted continuous function spaces for the integral operator as follows:

$$T_\xi x(t) = -\int_t^\infty e^{\zeta_1(t-s)} F(x(s), y(s)) ds,$$

$$T_\xi y(t) = \xi e^{\zeta_2 t} + \int_0^t e^{\zeta_2(t-s)} F(x(s), y(s)) ds.$$

From this admissibility we can prove the existence theorem for the special simultaneous differential equation. This existence theorem can be applied for the second order differential equation,

$$u'' = f(u, u') = \frac{kT(u)(1+u'^2)^{3/2}}{v\gamma} ln(\frac{P_0(u)}{P_c}).$$

The solution of this equation is one of the stationary solution for Mullins Equation.

References

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