The Hermit-quadratic and Hermit-cubic Finite-element Approximations to the Non-linear Problems

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For the inviscid Burger's equation written in the flux form the difference schemes in Hermitian finite-element spaces $V^{(2)}$ and $V^{(3)}$ are constructed. Several examples of solutions are included. Let us consider the equation: $\frac{\partial \phi}{\partial t} + \frac{1}{2}\frac{\partial \phi^2}{\partial x} = 0$. Applying the finite-difference method, it is usually approximated with the differential-difference equation: $\frac{d\phi_i}{dt} + \frac{1}{2}\left(\frac{\phi_{i+1}^2 - \phi_{i-1}^2}{2\Delta x}\right) = 0$. Using the finite-element method in Lagrangian space with *chapeau* expansion functions $\varphi_i(x)$ one can obtain more complicated non-linear difference scheme: $\frac{d}{dt}\left(\frac{\phi_{i-1}+4\phi_i+\phi_{i+1}}{6}\right) + \frac{1}{\Delta x}\left[-\left(\phi_i^s + 2\phi_{i-1}^s\right)\phi_{i-1}^{s+1} - \left(\phi_{i-1}^s - \phi_{i+1}^s\right)\phi_i^{s+1} + \left(\phi_i^s + 2\phi_{i+1}^s\right)\phi_{i+1}^{s+1}\right] = 0$, where $s-$ the number of iteration on the upper time level. The problem will increase, if we use the Hermite-quadratic or Hermite-cubic finite-element approximations

$$\phi(x,t) = \sum_i \left[\phi_i^f(t)\psi_i^f(x) + \phi_i^d(t)\psi_i^d(x)\right] \quad \psi_i^f(x), \psi_i^d(x) \text{ - quadratic or cubic functions}$$

The finite-element schemes for the term $\frac{1}{2}\frac{\partial \phi^2}{\partial x}$ have very complicated forms. They are solved iteratively. They will be discussed during the presentation.

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