Bifurcation Problem for a Semilinear Elliptic Equation Arising in Population Dynamics, Having Nonlinear Boundary Conditions

KENICHIRO UMEZU
Faculty of Engineering
Maebashi Institute of Technology, Japan
E-mail: ken@maebashi-it.ac.jp

In this talk we discuss a bifurcation problem of positive solutions for the following semilinear elliptic boundary value problem with a nonlinear boundary condition.

$$-\Delta u = \lambda(m(x) - au)u \text{ in } D, \qquad \frac{\partial u}{\partial \mathbf{n}} = b(x)g(u) \text{ on } \partial D,$$

where $D \subset \mathbf{R}^N$, $N \geq 2$, is a bounded domain with smooth boundary ∂D, $\lambda > 0$ is a parameter, $m \in C^\theta(\overline{D})$, $0 < \theta < 1$, may change its sign, $a > 0$ is a constant, $b \in C^{1+\theta}(\partial D)$ satisfying $b \geq 0$, $g \in C^{1+\theta}([0, \delta])$ for any $\delta > 0$ such that $g(0) = 0$, and $\mathbf{n}$ is the unit outer normal to ∂D.

This talk is devoted to the study of the existence and qualitative behavior of positive solutions when $\lambda \to +0$. In particular, we investigate *bifurcation to the right* emanating from the origin $(\lambda, u) = (0, 0)$, meaning that there exist positive solutions $(\lambda_j, u_j) \in (0, \infty) \times C^{2+\theta}(\overline{D})$ satisfying $(\lambda_j, u_j) \to 0$. In the homogeneous Neumann case $\partial u/\partial \mathbf{n} = 0$, it is known that $(\lambda, u) = (0, 0)$ is a bifurcation point to the right if, and only if, $\int_D m dx = 0$. Our main result generalizes this result to the nonlinear case. The global structure of bifurcation curves is also considered.