
Existence of Renormalized Solutions for Reaction Diffusion Equations

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We introduce the notion of renormalized solutions and consider existence of such solutions for a reaction diffusion equation which has extremely large initial data.

Let Ω be a bounded domain in \mathbb{R}^N , $N \geq 1$, with a Lipschitz boundary $\partial\Omega$ whenever $N \geq 2$, and let $T > 0$. We consider the following reaction diffusion equation

$$(RD) \quad \begin{cases} \frac{\partial u}{\partial t} - \Delta u = u^2 & \text{in } Q = (0, T] \times \Omega, \\ u = 0 & \text{on } \Sigma = [0, T] \times \partial\Omega, \\ u(0) = g & \text{in } \Omega, \end{cases}$$

where $g : \Omega \rightarrow \mathbb{R}$ is a given nonnegative and nontrivial function.

Reaction diffusion equations are related to many equations arising from mathematical biology, however, we have not yet obtained global existence results for reaction diffusion equations in the case of extremely large initial data in an appropriate sense. When $1 \leq N \leq 2$, in general, the local solution of (RD) blows up in finite time while the initial value g is nontrivial, and when $N > 2$, if g is small then (RD) has a global solution decaying to 0 as $t \rightarrow \infty$. On the other hand, if g is large enough then the local solution blows up in finite time. In the case of large initial data, it is a good way to utilize the concept of renormalized solutions which are defined by truncated functions. We there introduce the notion of renormalized solutions and prove existence of such solutions for the reaction diffusion equation (RD) which has extremely large initial data g .

References

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