Semilinear Elliptic Equations in Unbounded Domain

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The existence of a bounded, radial solution in unbounded domain $\Omega = \mathbb{R}^n \setminus B(0,1)$ with $n \geq 3$ of the nonlinear elliptic problem

$$\begin{aligned} &\Delta u = f(x,u) \ \text{ in } \ \Omega, \\ &u = 0 \ \text{ on } \ \partial\Omega \end{aligned} \tag{1}$$

is proved under some asymptotic and sign condition on f. Since the above problem is at resonance (one has one-dimensional kernel spanned by $u_0(x) = 1 - |x|^{2-n}$) we perturb the linear operator to get $\Delta - \lambda I$ for $\lambda > 0$ thus reducing perturbed equation to the operator equation $u = (\Delta - \lambda I)^{-1} N u$ in the space of bounded and continuous function (where N stands for Nemytski operator generated by f). To this one can apply any suitable fixed point theorem. Having obtained solution of the perturbed equation we show the existence of some sequence u_{λ_n} convergent to the Hölder continuous solution of the resonant problem (assuming f to be Hölder continuous with respect to x). In the case of nonlinearity f depending radially on $|x|$ we prove analogous existence result, reducing equation (1) to BVP for ODE on half-line. Under stronger sign assumption existence of a positive solution is also obtained.