Connections Between the Convective Diffusion Equation and the Forced Burgers Equation

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The convective diffusion equation with drift $b(x)$ and indefinite weight $r(x)$:

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial x}\left[a\frac{\partial \phi}{\partial x} - b(x)\phi\right] + \lambda r(x)\phi, \tag{1}$$

is introduced as a model for population dispersal. Strong connections between Eq.(1) and the forced Burgers equation with positive frequency $(m \geq 0)$:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - u\frac{\partial u}{\partial x} + mu + k(x), \tag{2}$$

are established through the Hopf-Cole transformation. Eq.(2) is a prime prototype of the large class of quasilinear parabolic equations given by

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial (f(v))}{\partial x} + g(v) + h(x). \tag{3}$$

A compact attractor and an inertial manifold for the forced Burgers equation are shown to exist via the Kwak transformation. Consequently, existence of an inertial manifold for the convective diffusion equation is guaranteed. Eq.(2) can be interpreted as the velocity field precursed by Eq.(1). Therefore, the dynamics exhibited by the population density in Eq.(1) show their effects on the velocity expressed in Eq.(2). Numerical results illustrating some aspects of the previous connections are obtained by using a pseudospectral method.