Solutions with Internal Jump for an Autonomous Elliptic System of Bistable Type

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We consider the following system of semilinear elliptic equations

$$\begin{cases} -\varepsilon^2 \Delta u &= f(u) - v & \text{in} & \Omega; \\ \gamma v - \Delta v &= \delta u & \text{in} & \Omega; \\ u &= v = 0 & \text{on} & \partial\Omega. \end{cases} \tag{1}$$

We assume Ω to be a smooth bounded domain in $\mathbb{R}^N$, with $N \geq 1$, while γ is larger than the first eigenvalue of $-\Delta$ on Ω subjected to homogeneous Dirichlet boundary conditions. We take $\varepsilon > 0$ and $\delta \geq 0$ as parameters. The nonlinearity we assume for simplicity to be $f(u) = u(u-1)(a-u)$ with $0 < a < 1/2$, although other more general nonlinearities can also be treated. We observe that the system is coupled in a noncooperative way, and hence is not order preserving. This leads to a richer solution structure. In particular for small $\delta \geq 0$ the solutions to (1) are similar to the solutions to the scalar equation

$$\begin{cases} -\varepsilon^2 \Delta u &= f(u) & \text{in} & \Omega; \\ u &= 0 & \text{on} & \partial\Omega, \end{cases} \tag{2}$$

for which it is known that, under certain assumptions on Ω and for ε small, there exist only two nontrivial solutions. We shall present results which show how this simple solution structure becomes more complex as δ increases.