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## Symmetry Breaking Phenomena in an Optimization Problem for Some Nonlinear Elliptic Equation

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In this talk, we consider an optimization problem associated with the following nonlinear elliptic boundary value problem:

$$-\Delta u + \lambda u^p = \chi_D, u > 0, \quad (x \in \Omega), \quad u = 0 \quad (x \in \partial\Omega),$$

where  $\Omega$  is a bounded domain in  $\mathbf{R}^n$  with smooth boundary,  $\lambda > 0$ , and  $1 \leq p < (n+2)/(n-2)$  if  $n \geq 3$  and  $1 \leq p < +\infty$  if  $n = 1, 2$ . Here,  $D$  is a measurable subset of  $\Omega$  which belongs to the class:  $\mathcal{C}_\beta = \{D \subset \Omega \mid |D| = \beta\}$  for the prescribed  $\beta \in (0, |\Omega|)$ . It is well-known that for any  $D \in \mathcal{C}_\beta$ , there exists a unique solution  $u \in H_0^1(\Omega)$ , we denote it by  $u_D$ , to the nonlinear boundary value problem above and the solution  $u_D$  is obtained as a global minimizer of the functional:

$$J_D(v) = \frac{1}{2} \int_{\Omega} |\nabla v|^2 dx + \frac{\lambda}{p+1} |v|^{p+1} dx - \int_{\Omega} \chi_D v dx$$

on  $H_0^1(\Omega)$ . Now, we consider the optimization problem:  $E_{\beta, \Omega} = \inf_{D \in \mathcal{C}_\beta} J_D(u_D)$ . If  $E_{\beta, \Omega}$  is attained by  $D^* \in \mathcal{C}_\beta$ , then we call  $D^*$  an optimal configuration. In this talk, we present our recent results on the existence, uniqueness and non-uniqueness, symmetry preserving and symmetry breaking phenomena of the optimal configuration  $D^*$  to this optimization problem in various settings. When  $\lambda = 0$ , similar results has been obtained by Cuccu and Porru in [1]. We extend their ideas to be adapted to our setting and overcome some difficulty to show symmetry breaking in the nonlinear case. We should also mention that such symmetry breaking phenomena was first discovered in the optimization of the first Dirichlet eigenvalue to the Schrödinger operator in [2].

### References

- [1]. F. Cuccu, G. Porru, Optimization in a problem of heat conduction, preprint 2001.
- [2]. S. Chanillo, D. Griser, M. Imai, K. Kurata, I. Ohnishi, Symmetry breaking and other phenomena in the optimization of eigenvalues for composite membranes. *Comm. Math. Phys.*, **214** (2000), 315–337.