Mathematical Analysis of the Quasilinear Effects in a Hyperbolic Model of Blood Flow Through Compliant Axi-symmetric Vessels

Eun Heui Kim
Department of Mathematics
California State University, USA
E-mail: ekim4@csulb.edu

This is a joint work with S. Canic. We present a mathematical analysis of the quasilinear effects arising in a hyperbolic system of partial differential equations modeling blood flow through large compliant vessels. The equations are derived using asymptotic reduction of the incompressible Navier-Stokes equations in narrow, long channels. We discuss the estimates on the initial and boundary data which imply strict hyperbolicity in the region of smooth flow and outline a proof of a general theorem which provides conditions under which an initial-boundary value problem for a quasilinear hyperbolic system admits a smooth solution. Using this result we show that pulsatile flow boundary data always give rise to shock formation (high gradients in the velocity and inner vessel radius). We estimate the time and the location of the first shock formation and show that in a healthy individual, shocks form well outside the physiologically interesting region (2.8 meters downstream from the inlet boundary). We also present numerical results on the model problem.