Existence and Uniqueness for a Size Structured Population Model with Nonlinear Growth Rate

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We give the existence and uniqueness of the solution to a size structured population model with the growth rate depending on the individual's size and its total population at each time. The model comes from the population models of plants in forests or plantations and described by the following initial boundary value problem with nonlocal boundary condition:

$$\text{(NSDP)} \begin{cases} u_t + (V(x, P(t))u)_x = G(u(\cdot,t))(x), \quad x \in [0,l), \ 0 \le t \le T, \\ V(0, P(t))u(0,t) = C(t) + F(u(\cdot,t)), \quad 0 \le t \le T, \\ u(x,0) = u_0(x), \quad x \in [0,l). \\ P(t) = \int_0^l u(x,t)dx \end{cases} \tag{1}$$

The unknown function $u(x,t)$ stands for the density of population of size x at time t and $P(t) = \int_0^l u(x,t)dx$ represents the total population, where $0 < l \le \infty$ is the maximum size. The function $V(x, P(t))$ is the nonlinear growth rate function depending on the size x and the total population $P(t)$ at each time t. The functions F and G correspond to the birth and aging functions respectively and the function C represents the inflow of zero-size individuals (i.e. newborns) in the population from an external source (for example, seeds carried by the wind or placed in a plantation).

Our results are the generalizations of the ones due to A. Calsina and J. Saldaña in 1995, where they treated the Gurtin-MacCamy type model, i.e., F and G have the following forms :

$$F(u(\cdot,t)) = \int_0^l \beta(x, P(t))u(x,t)dx, \quad G(u(\cdot,t))(x) = -m(x, P(t))u(x,t). \tag{2}$$

Their arguments strictly depend on these special forms. For a general model as above, we need a different method. We emphasize that for the uniqueness, some additional assumptions are needed compared with the existence result and it is not known whether the uniqueness is obtained under the same conditions as in the existence result.