
Pointwise Error Estimate for an Elliptic System of Quasi-Variational Inequalities

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A number of results on pointwise error estimates for the classical obstacle problem in particular, and variational inequalities (VI) in general, have been achieved in the last three decades, where the most significant ones are due to Nitsche (1977), C. Baiocchi (1977), P. Cortey-Dumont (1985), R.H. Nochetto (1988). However, very little is known on the subject when it comes to quasi-variational inequalities (QVIs) In the present work we investigate the approximation in the L^∞ norm for the following system of QVIs: find $(u^1, \dots, u^M) \in (H_0^1(\Omega))^M$ such that

$$\begin{cases} a^i(u^i, v - u^i) \geq (f^i, v - u^i) \quad \forall v \in H_0^1(\Omega), \quad v \leq k + u^{i+1} \\ u^i \leq k + u^{i+1}; \quad \text{with} \quad u^{M+1} = u^1 \end{cases} \quad (1)$$

where Ω is a bounded smooth domain of R^N , $N \geq 1$, $a^i(\cdot, \cdot)$ are continuous bilinear forms associated with second order elliptic operators, (\cdot, \cdot) denotes the standard inner product in $L^2(\Omega)$, f^i are regular functions, and k is a positive number. The system under consideration plays a key role in solving Hamilton-Jacobi-Bellman equations (HJB) studied by P.L. Lions and J.L Menaldi (1979).

We show that the piecewise linear approximation applied to this system is quasi-optimally accurate in $L^\infty(\Omega)$. Our study includes both the coercive and noncoercive problems, where different approaches are respectively developed and analyzed. As a consequence of our main result, we also derive a pointwise error estimate for the corresponding HJB equations.

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