
Finite Element Approximation to Evolution Problems in Mixed Form

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This study is a first step towards the analysis of a modification of the immersed boundary method proposed by Peskin.

Mixed finite elements are often used in engineering applications and their analysis has been considered in several papers, starting from the 70's, mainly for the approximation of steady source problems. When a finite element approximation to a mixed is considered, it is well-known that the necessary and sufficient condition for the well-posedness, stability and convergence of the scheme (for any given data) is that two inf-sup constants are bounded below away from zero independently of the meshsize parameter. In the 80's the use of mixed finite elements has been considered also for the approximation of eigenvalue problems and only fairly recently it has been understood that the inf-sup conditions are not the main assumptions in this context.

We consider the finite element approximation of evolution problems in mixed form. There is few mathematical literature on this field, mainly related to the approximation of the heat equation by means of Raviart-Thomas elements, even though mixed finite element schemes have been extensively used for the approximation of evolution problems, in particular in fluid-dynamic applications.

We will give theorems, stating sufficient conditions for the good approximation of the problems under consideration and report on numerical tests confirming our theory. In particular, we present an example of discretization of the heat equation for which the standard inf-sup conditions are satisfied (hence the corresponding steady Poisson problem is well approximated) but which does not provide a good scheme for the evolution heat equation.