Table of Contents

Abstracts of Plenary Talks

Abstracts of Invited Talks

Abstracts of Contributed Talks

Abstracts of Plenary Talks

Analytic Consequences of Incompressibility

Stuart S. Antman
Institute for Physical Science & Technology
University of Maryland, USA
E-mail: ssa@math.umd.edu

Many deformable solids, like rubber and living tissue, are effectively incompressible. This means that the Jacobian determinants of all their deformations must be everywhere equal to 1. This talk will show by simple examples that the quasilinear partial differential equations governing the motion of incompressible bodies are much more complicated than those governing the motion of compressible bodies (whose deformations need only preserve orientation), but have solutions with far more regularity. The role of incompressibility will be related to the question of constructing invariant dissipative mechanisms for hyperbolic conservation laws.

Numerical Relativity

Douglas N. Arnold
Institute for Mathematics and Its Applications
University of Minnesota, USA
E-mail: arnold@ima.umn.edu

The numerical solution of Einstein's equations of general relativity promises to become one of the most potent tools for understanding the complex behavior of strong dynamical gravitational fields. In particular, the tremendous international investment in gravitational wave observatories–which promise to open our first window on the universe looking outside electromagnetic spectrum–depends on our learning to interpret gravitational signals radiated from violent cosmic events, such as black hole collisions. This will require accurate numerical simulation of Einstein's equations of general relativity.

The Einstein equations form a system of ten second order nonlinear partial differential equations in four-dimensional spacetime which, while having a very elegant and fundamental geometric character, are extremely complex. Their numerical solution will surely require new ideas, as well as the application of state-of-the-art numerical methods from other areas of computational physics. Despite recent progress in various directions, the development of accurate, efficient, and validated algorithms for Einstein's equations remains a grand challenge. This talk aims to introduce some of the scientific, mathematical, and computational problems involved in the burgeoning field of numerical relativity, and to suggest directions of future research.

Compatibility Condition for Microstructures

John M. Ball
Mathematical Institute
University of Oxford, UK
E-mail: ball@maths.ox.ac.uk

The talk will cover two topics related to the study of microstructure arising from solid phase transformations. The first concerns work with C. Carstensen on generalizations of Hadamard's jump condition for Lipschitz mappings. This involves in particular defining limiting sets of gradients of a Lipschitz mapping or $W^{1,\infty}$ gradient Young measure from either side of a surface at an arbitrary point. The second is joint work with an electron microscopist D. Schryvers (Antwerp) and shows how an understanding of compatibility can be of use in constructing plausible scenarios for macrotwin formation in NiAl alloys.

Stability of Approximate Solutions for Systems of Conservation Laws

Alberto Bressan
Functional Analysis and Applications Sector
International School for Advanced Studies, Italy
E-mail: bressan@sissa.it

Consider a strictly hyperbolic $n \times n$ system of conservation laws in one space dimension:

$$u_t + f(u)_x = 0\,. \tag{1}$$

For small BV initial data, the global existence, uniqueness and stability of entropy weak solutions is now well known.

Various methods for constructing approximate solutions have been considered in the literature. In particular:

(i) Vanishing viscosity

(ii) Relaxation approximations

(iii) Numerical schemes

The talk will address the problem of obtaining a priori BV bounds, stability and convergence results for these approximate solutions. We shall review the basic ideas in the proof valid for vanishing viscosity approximations, and discuss what can be done in the case of solutions computed by finite difference schemes.

On Some Equilibrium Problems

MICHEL CHIPOT
Angewandte Mathematik
Universität Zürich, Switzerland
E-mail: chipot@amath.unizh.ch

After having in [1] studied the equilibrium position of one ball rolling on an elastic membrane we wanted to investigate the case of several balls. However, the problem is not so easy to handle and to get some insight we turned first to the case of several disks rolling on a wire (see [2], [3]). We considered the case of disks different in size and weight, but to illustrate the surprising situations that can occur let us just mention what happens when one considers two identical disks (in size and weight). A first guess is to believe that these two disks will reach their equilibrium when they are sitting at the same level in the middle of the wire. This is indeed true for light disks. But when the weight of the disks increases they adopt a position slighty tilted and two symmetric equilibria can be reached that way. Thus, the increase in the tension of the wire rendered the equilibrium unstable. Some other surprising behaviours will be explained and several open problems will be pointed out.

References

[1]. J. Bemelmans and M. Chipot, On a variational problem for an elastic membrane supporting a heavy ball. *Calculus of Variations and PDE's*, **3** (1995), 272–298.

[2]. A. Aissani, M. Chipot and S. Fouad, On the deformation of an elastic wire by one or two heavy disks. *Archiv der Mathematik*, **76** (2001), 467-480.

[3]. M. Chipot. In preparation.

On the Recovery of a Surface with Prescribed First and Second Fundamental Forms

PHILIPPE G. CIARLET
Department of Mathematics, City University of Hong Kong and
Laboratoire d'Analyse Numérique, Université Pierre et Marie Curie, France
E-mail: mcpgc@cityu.edu.hk

The fundamental theorem of surface theory asserts that, if a field of positive definite symmetric matrices of order two and a field of symmetric matrices of order two together satisfy the Gauss and Codazzi-Mainardi equations in a connected and simply connected open subset of $\mathbb{R}^2$, then there exists a surface in $\mathbb{R}^3$ with these fields as its first and second fundamental forms (global existence theorem) and this surface is unique up to isometries in $\mathbb{R}^3$ (rigidity theorem).

The aim of this lecture, which is based on a recent joint work with François Larsonneur, is to provide a self-contained and essentially elementary proof of this theorem by showing how it can be established as a simple corollary of another well-known theorem of differential geometry, which asserts that, if the Riemann-Christoffel tensor associated with a field of positive definite symmetric matrices of order three vanishes in a connected and simply connected open subset of $\mathbb{R}^3$, then this field is the metric tensor field of an open set that can be isometrically imbedded in $\mathbb{R}^3$ (global existence theorem) and this open set is unique up to isometries in $\mathbb{R}^3$ (rigidity theorem).

A New Approach to the Riemann Problem for Hyperbolic Conservation Laws

CONSTANTINE M. DAFERMOS
Division of Applied Mathematics
Brown University, USA
E-mail: `dafermos@cfm.brown.edu`

The Riemann problem for hyperbolic systems of conservation laws plays a central role in the theory, because its solution describes both the local structure and the long time behavior of general solutions and serves, in addition, as the building block for solving the Cauchy problem by either the random choice method or the front tracking algorithm.

The classical approach for solving the Riemann problem pieces together shocks and rarefaction waves with the help of shock and rarefaction wave curves. More recently, the vanishing viscosity method has also been successfully employed. The lecture will discuss how the classical Riemann problem can be formulated and solved as a variational problem.

On the Numerical Simulation of Particulate Flow for Non-Newtonian Incompressible Viscous Fluids a la Bingham

ROLAND GLOWINSKI
Department of Mathematics
University of Houston, USA
E-mail: `roland@math.uh.edu`

These last years have seen a renewal of interest concerning the mathematical and computational aspects of incompressible viscous flow a la Bingham. Compared to Newtonian situations the main difficulty is related to the fact that the mathematical model contains a multivalued partial differential operator which is the sub-gradient of an L1-norm of the rate-of-strain tensor. Through an equivalent formulation involving a tensor-valued multiplier it has been known for years that the above difficulty is easily overcome from a computational point of view.

The main goal of this lecture is to discuss an operator splitting based methodology combining the above multiplier characterisation with further computational ingredients (such as L2-projection methods for the treatment of the incompressibility condition div u=0, and fictitious domain techniques to take care of moving boundaries) in order to address the direct numerical simulation of Bingham particulate flow, i.e., the flow of mixtures of Bingham fluids and solid particles (assumed to be rigid here).

Soft Ferromagnetic Thin Films

Robert V. Kohn
Courant Institute of Mathematical Sciences
New York University, USA
E-mail: kohn@cims.nyu.edu

Micromagnetics is a nonlocal, nonconvex variational problem. Its local minimizers represent the stable magnetization patterns of a ferromagnetic body under a specified applied field. The analysis of soft thin films is particularly rewarding because their patterns are readily observable. This problem is rich, experimentally and mathematically, because there are three interacting length scales: the thickness and diameter of the film, and the exchange length of the magnetic material. I will discuss recent work with DeSimone, Otto, and Mueller, which identifies a physically relevant thin film limit and derives, in this limit, a reduced variational problem. The reduced problem is degenerate but convex; as a result it determines some but not all features of the ground state magnetization pattern.

Mathematical and Numerical Analysis of Micro-Macro Simulations for Polymeric Fluid Flows

Claude Le Bris
Centre d'Enseignement et de Recherche en Mathématiques
École Nationale des Ponts et Chaussées, France
E-mail: lebris@cermics.enpc.fr

We shall review some recent contributions on the analysis of micro-macro models for polymeric fluid flows. The system of equations that needs to be simulated numerically is a coupled system consisting of the Navier-Stokes equations (at the macroscopic scale) together with some kinetic equations at the microscopic scale. The kinetic equations, of Fokker-Planck type, may be solved as such, or transformed into stochastic differential equations that are in turn discretized. Such a coupled system raises many questions, both of theoretical and numerical nature. Issues about the well-posedness of the system, as well as questions dealing with the convergence of finite elements methods in this framework will be dealt with. The talk will report on joint works with E. Cances, Y. Gati, B. Jourdain, T. Lelievre, PL. Lions.

Parareal in Time Simulation for Partial Differential Equations

Yvon Maday
Laboratoire d'Analyse Numérique
Université Pierre et Marie Curie, France
E-mail: `maday@ann.jussieu.fr`

Numerical simulation of time dependant problems on complex geometries is s till a challenging task. In a context of fast increasing in both the CPU power available on typical workstations and of the number of computers that can be connected through high speed networks, the difficulty resides rather in how to obtain "real time solutions" than in the amount of CPU power available (which becomes to exceed the needs). In this direction, domain decomposition and splitting techniques is interesting but not enough.

In this context, the "parare al" algorithm that parallelize **in the time direction** the work required to solve the evolution equations will be presented on many illustrative examples. This method is based on the alternative use of coarse global sequential solvers with fine local parallel ones.

A rewritting of the time scheme in a matrix form can be presented that allows for an application to control of phenomenon governed by parabolic type equations.

Estimates for Elliptic Systems for Composite Material

Louis Nirenberg
Courant Institute of Mathematical Sciences
New York University, USA
E-mail: `nirenl@courant.nyu.edu`

In a bounded domain D in $\mathbb{R}^n$, we consider a composite media whose physical characteristics are smooth in the closures of subdomains D_m but possibly discontinuous across their boundaries. The properties of the media are described in terms of a linear second order elliptic system. The coefficients are smooth in the closure of each D_m but not across their boundaries. Under suitable conditions we obtain bounds on the first derivatives of the solution, and their Hölder continuity, in the closure of each D_m - provided we stay away from the boundary of D. The estimates depend on the number of domains but are independent of how close they are to each other; they may even touch.

Mathematical Modeling and Numerical Simulation of the Cardiovascular System

Alfio Quarteroni
Departement de Mathematiques
École Polytechnique Fédérale de Lausanne, Switzerland
E-mail: `alfio.quarteroni@epfl.ch`

The interest in the use of mathematical modeling and numerical simulation in the study of the cardiovascular system (and its inherent pathologies) has greatly increased in the past few years. Blood flow interacts mechanically and chemically with vessel walls producing a complex fluid-structure interaction problem, which is practically impossible to simulate in its entirety.

Several reduced models have been developed which may give a reasonable approximation of averaged quantities, such as mean flow rate and pressure, in different sections of the cardiovascular system. They are, however, unable to provide the details often needed for understanding a local behavior, such as, e.g., the effect on the shear stress distribution due a modification in the blood flow consequent to a partial stenosis.

In this talk we address some mathematical issues arising from the modeling of the cardiovascular system through problems of different complexity. The most complex model is based on the coupling of the Navier-Stokes equations with structural models for the vessel walls. An intermediate model is derived from integrating these equations on a section of a vessel geometry, and it is formed by a system of hyperbolic equations for the evolution of mean pressure and flow rate. An even simpler model that will be considered is based on the solution of a system of ordinary differential equations which describe electrical networks.

The derivation of these models will be presented together with schemes for their numerical solution. Furthermore, we will specifically address the coupling problem, analyzing different possible strategies. These techniques may be extended by including models for chemical transport. Some results in this direction are already available and will be presented.

The previous multi-scale approach looks a viable solution to obtain detailed numerical simulation of sections of the cardiovascular apparatus, while properly accounting for the presence of the global system. We expect that this technique will open new possibilities for the use of numerical modeling for medical research.

Rapid Solution of Evolution Equations in High Dimensions

CHRISTOPH SCHWAB
Seminar fur Angewandte Mathematik
ETH Zürich-Zentrum, Switzerland
E-mail: schwab@sam.math.ethz.ch

We analyze a discontinuous Galerkin time-stepping procedure for the numerical solution of nonlinear evolution equations in Hilbert or (reflexive) Banach spaces. Time steps and orders can be varied. For solutions which exhibit piecewise Gevrey regularity of index δ in time (expressed by suitable countably normed spaces) we show that exponential convergence of order $O(\exp(-bN^{1/(\delta+1)}))$ can be achieved, where N denotes the number of spatial problems to be solved.

Applications include the evolution of 'Perfect incompressible fluids' where $\delta = 3$ and abstract parabolic equations where $\delta = 1$.

In the pricing of options on baskets of d underlyings or on indices, the spatial domains of the parabolic evolution problems arising in the Black-Scholes setting are often hypercubes in R^d with d, the number of underlyings, large (typically $5 \leq d \leq 50$). We show how the DG time-stepping scheme can be combined with a sparse tensor product wavelet discretization in d dimensions to give an overall scheme with convergence rate $O(h^p|\log h|^{(d-1)/2})$ with work of order $O(h^{-1}|\log h|^{d+2})$. Numerical experiments in dimensions $d <= 20$ on a personal computer confirm the theoretical results.

This is joint work with RISKLAB of ETH Zürich and with T. von Petersdorff of University of Maryland, College Park, USA.

Scientific Computations Related to the Riemann Hypothesis

RICHARD S. VARGA
Institute for Computational Mathematics
Kent State University, USA
E-mail: varga@mcs.kent.edu

It is well known that the Riemann Hypothesis is equivalent to the statement that all zeros of the Riemann function $\Xi(x)$ are real. This function can be expressed as the integral

$$\Xi(\frac{x}{2})/8 = \int_o^\infty \Phi(u)\cos(xu)du \qquad (x \in \mathbb{C}),$$

where

$$\Phi(u) := \sum_{n=1}^{\infty}(2\pi^2 n^4 e^{9u} - 3\pi n^2 e^{5u})\exp(-\pi n^2 e^{4u}) \quad (0 \leq u < \infty).$$

Pólya considered the more general trigonometric integral

$$H_t(x) := \int_o^\infty e^{tu^2}\Phi(u)\cos(xu)du \qquad (t \in \mathbb{R}; x \in \mathbb{C}),$$

where we see that H_o and the $\Xi-$ function are related through

$$H_o(x) = \Xi(\frac{x}{2})/8,$$

so that the Riemann Hypothesis is also equivalent to the statement that all zeros of H_o are real.

Based on the work of de Bruijn(1950), C. Newman(1976) showed that there is a real constant Λ, with $-\infty < \Lambda \leq 1/2$, such that H_t has only real zeros if and only if $t \geq \Lambda$. This constant Λ is now called the **de Bruijn-Newman constant** in the literature and it follows that

the Riemann Hypothesis is true if and only if $\Lambda \leq 0$.

Lower bounds for Λ, such as

$$-5.895 \cdot 10^{-9} < \Lambda \quad and \quad -2.7 \cdot 10^{-9} < \Lambda,$$

have quite recently been determined, and we show how analysis techniques, combined with known numerical values for some zeros of $\Xi(x)$, produce these lower bounds above. (Curiously, no upper bound for Λ, better than de Bruijn's original $\Lambda \leq 1/2$, has been established.)

Analysis and Control of some Models for Fluid-structure Interaction

ENRIQUE ZUAZUA
Departamento de Matemáticas
Universidad Autónoma, Madrid, Spain
E-mail: enrique.zuazua@uam.es

In this lecture we shall discuss some simplified models for fluid-structure interaction. From a mathematical point of view these models are characterized by the coupling of two different components in which parabolic and hyperbolic equations are coupled, through, possibly, finite dimensional differential equations for solid masses.

First we shall analyse a one-dimensional model for the interaction between a fluid and a solid mass. The fluid is governed by the viscous Burgers equation and the solid mass, which shares the velocity of the fluid, is accelerated by the difference of pressure at both sides of it. We describe the asymptotic behavior of solutions for integrable data using energy estimates and scaling techniques. We prove that the asymptotic profile of the fluid is a self-similar solution of the Burgers equation with an appropriate mass and we describe the parabolic trajectory of the solid mass. We also consider the case of a finite number of masses and we show that they may not collide in finite time.

We then consider a linearized model coupling a wave with a heat equation. We dicuss the main properties of this model in what concerns, well-posedness and regularizing effects. A comparison with the classical system of thermoelasticity will be given empashizing the new aspects related to the fact that the two equations do not hold in the same domain but rather in two domains separated by an interface. The rate of decay and controllability of this system will also be discussed. As we shall see, the nature of the hyperbolic-parabolic coupling makes this issue complex and very much dependent on the geometry of the two domains involved in the model.

Abstracts of Invited Talks

Nonsymmetric Positive Solutions for Symmetric Dirichlet Elliptic Problems

DAOMIN CAO
Institute of Applied Mathematics, Academy of Maths & System Sciences
The Chinese Academy of Sciences, P. R. China
E-mail: cao@amath6.amt.ac.cn

In this talk the speaker will present some results on the existence of non-radially symmetric positive solutions of the following radially symmetric problems when ε is small positive number:

$$\begin{cases} -\Delta u = |x|^\tau u^{\frac{N+2}{N-2}-\varepsilon} & x \in \Omega, \\ u > 0 & x \in \Omega, \\ u = 0 & x \in \partial\Omega, \end{cases} \tag{1}$$

where $N \geq 3, \Omega$ be the unit ball in $\mathbb{R}^N$ centered at the origin and $\tau > 0$ be a given number. and

$$\begin{cases} -\Delta u + (\frac{1}{\varepsilon} - h(x))u = (1 - f(x))u^p, \quad \text{in} \quad \mathbb{R}^N, \\ \quad u > 0, \quad \text{in} \quad \mathbb{R}^N, \\ \quad u \in H^1(\mathbb{R}^N), \end{cases} \tag{2}$$

where $h(x)$ and $f(x)$ are nonnegative radially symmetric functions in $L^\infty(\mathbb{R}^N)$, $h(x)$ and $f(x)$ have compact support in $\mathbb{R}^N$, $f(x) \leq 1$ for all $x \in \mathbb{R}^N$, $1 < p < +\infty$ for $N = 1, 2$, $1 < p < \frac{N+2}{N-2}$.

Implicit PDEs and Existence of Minimizers in the Calculus of Variations

BERNARD DACOROGNA
Departement de Mathematiques
École Polytechnique Fédérale de Lausanne, Switzerland
E-mail: bernard.dacorogna@dma.epfl.ch

We discuss the existence of Lipschitz solutions of implicit equations of the type

$$\begin{cases} F\left(x, u\left(x\right), Du\left(x\right)\right) = 0, \text{ a.e. in } \Omega \\ u = \varphi, \text{ on } \partial\Omega \end{cases}$$

where $\Omega \subset \mathbb{R}^n$ is an open set, $u : \Omega \to \mathbb{R}^m$ is vector valued , $F : \Omega \times \mathbb{R}^m \times \mathbb{R}^{m\times n} \to \mathbb{R}$ is continuous and φ is a given Lipschitz map.

We then show how this analysis applies to minimization problems of the form

$$\inf \left\{ \int_\Omega f\left(x, u\left(x\right), Du\left(x\right)\right) dx : u = \varphi \text{ on } \partial\Omega \right\}.$$

Fredholm Type Results for Nonlinear Operators

George Dinca
Faculty of Mathematics
University of Bucharest, Romania
E-mail: `dinca@cnfis.ro`

Consider an operator of the form $(\lambda T - S)$ where T and S are (generally nonlinear) operators from a Banach space X into Banach space Y, and T is invertible. The problem is then, to characterize those $\lambda \neq 0$ for which $(\lambda T - S)$ is surjective. To do so, two different methods are used.

The first one reduces the problem of the existence of a solution for the equation $(\lambda T - S)x = f$, to that of the surjectivity of the operator $(I - F)$, where

$$F \,:\, Y \longrightarrow Y, \quad Fy = S\big[T^{-1}\big(\frac{1}{\lambda}y\big)\big].$$

The arguments come from the spectral theory of nonlinear operators.

The second method reduces the same problem to that of the existence of a fixed point for the operator

$$A \,:\, X \longrightarrow X, \quad Ax = T^{-1}\big[\frac{1}{\lambda}(Sx + f)\big],$$

and the arguments come from the Leray-Schauder degree theory.

Some applications concerning the surjectivity for operators of the form $(\lambda J_\phi - S)$, where J_ϕ is a duality mapping, are considered. In particular, surjectivity results for $\lambda(-\Delta_p) - N_f$, where

$$\Delta_p u = \text{div}\ \big(|\,\nabla u\,|^{p-2}\nabla u\big)$$

is the so-called p-Laplacian, and

$$(N_f u)(x) = f(x, u(x)))$$

is the Nemytskii operator, are obtained.

Asymptotic Behaviour of a Bingham Fluid in Thin Layers

Srinivasan Kesavan
The Institute of Mathematical Sciences
India
E-mail: `kesh@imsc.ernet.in`

A Bingham fluid is a visco-plastic medium obeying the general laws of continuum mechanics and has a special nonlinear constitutive law. It is a non-Newtonian fluid which moves like a rigid body when a certain function of the stress tensor is below a certain threshold (sometimes called the yield stress). Beyond this yield stress, the fluid obeys a nonlinear constitutive law. In this paper, a nonlinear stationary model, in the form of a variational inequality giving the velocity and the pressure, is considered in a thin layer represented by the open set $(0,1) \times (0,\varepsilon)$ in the plane, where ε is a small parameter tending to zero. The original problem is then transformed into one posed over a fixed reference domain $(0,1) \times (0,1)$ thus bringing out the dependence on ε explicitly. The limit problem satisfied by the limits of the transformed variables as $\varepsilon \to 0$ is then studied. To do this we need to study the properties of a function space of Sobolev type. It is shown that the limit problem is well-posed in this function space. Finally, the differential equation satisfied by the limit variables in the 'non-rigid zone' is obtained and is compared with a one-dimensional model proposed in the engineering literature.

Partial Regularity of Local Minimizers

JAN KRISTENSEN
Department of Mathematics
Heriot-Watt University, UK
E-mail: `j.kristensen@ma.hw.ac.uk`

A strongly quasiconvex integral

$$I(u) = \int_{\Omega} F(\nabla u(x))\, dx$$

defined on suitable Sobolev maps $u\colon \Omega \subset \mathbb{R}^n \to \mathbb{R}^N$ can admit local minimizers (in various metrics) that are not global minimizers. In 1986 L.C. Evans showed that global minimizers $\bar{u}$ of $I(u)$ are partially regular: $\bar{u}$ is C^1 in an open subset of full n-dimensional measure in Ω. Recently, S. Müller and V. Šverák showed that $I(u)$ can admit extremals (i.e., weak solutions to the Euler-Lagrange equation: $\mathrm{div} F'(\nabla u) = 0$) that are Lipschitz, but nowhere C^1. In this talk I discuss the situation for local minimizers with respect to some natural metrics. The talk is based on joint work with Ali Taheri.

An Asymptotically Periodic Schrödinger Equation with Indefinite Linear Part

GONGBAO LI[♯] AND ANDRZEJ SZULKIN[†]
[♯] *Wuhan Institute of Physics and Mathematics, The Chinese Academy of Sciences, P. R. China*
[†] *Department of Mathematics, Stockholm University, Sweden*
E-mail: [♯]`ligb@wipm.whcnc.ac.cn`

We consider the Schrödinger equation $-\Delta u + V(x)u = f(x,u)$, where V is periodic and f asymptotically periodic in the x-variables, 0 is in a spectral gap of $-\Delta + V$ and f is either asymptotically linear or superlinear as $|u| \to \infty$. We show that this equation has a solution $u \in H^1(\mathbb{R}^N), u \neq 0$.

The first author was supported in part by NSFC and Academy of Finland. The second author was supported in part by the Swedish Natural Science Research Council.

Hydrodynamic Limit of the Boltzmann Equations

Nader Masmoudi
Courant Institute of Mathematical Sciences
New York University, USA
E-mail: masmoudi@courant.nyu.edu

We will overview different recent results on the subject. Indeed, after the work of C. Bardos, F. Golse and D. Levermore of 1989 a lot of difficulties were left open about a rigourous derivation and we will overview how some of them were solved. Then, we will explain more the case of a bounded domain. In particular, we will consider the Boltzmann equation in a bounded domain with different types of (kinetic) boundary conditions and derive the Stokes-Fourier system with different type of (fluid) boundary conditions when the main free path goes to zero.

Free-boundary Problems for the Ginzburg-Landau Model

Sylvia Serfaty
Courant Institute of Mathematical Sciences
New York University, USA
E-mail: serfaty@courant.nyu.edu

We will present obtained jointly with E. Sandier on the Ginzburg-Landau model of superconductivity. In the asymptotic limit of “large kappa”, we prove a Gamma-convergence result which shows that minimizers of the Ginzburg-Landau energy converge (in the sense of convergence of the vortex-density) to the solution of a simple obstacle problem. Free-boundary type limiting problems are also derived for nonminimizing critical points.

On Generalized Navier Boundary Condition for Contact Line Hydrodynamics of Immiscible Flows

Xiao-Ping Wang
Department of Mathematics
Hong Kong University of Science and Technology, Hong Kong
E-mail: mawang@ust.hk

We give a continuum formulation of the immiscible flow hydrodynamics comprising of the Navier-Stokes equation, the Cahn-Hilliard interfacial free energy and the generalized Navier boundary condition (GNBC). In GNBC, the amount of slipping is proportional to

the sum of tangential viscous stress and stress arising from the deviation of the fluid-fluid interface from its static configuration. Numerical solutions of our hydrodynamic model yield interfacial profile and velocity variations matching those from the molecular dynamic simulations at the vicinity of the contact line. This is a joint work with T.Z. Qian and P. Sheng.

Kinetic Theory for Inertial Suspensions

JONATHAN WYLIE
Department of Mathematics
City University of Hong Kong, Hong Kong
E-mail: wylie@math.cityu.edu.hk

We consider the averaged flow properties of a suspension in which the Reynolds number based on the particle diameter is finite so that the inertia of the fluid phase is important. When the inertia of the particles is sufficiently large, their trajectories between successive particle collisions are only weakly affected by the interstitial fluid. If the particle collisions are nearly elastic the particle velocity distribution is close to an isotropic Maxwellian. The rheological properties of the suspension can then be determined using kinetic theory, provided that one knows the granular temperature (energy contained in the particle velocity fluctuations). This energy results from a balance of the shear work with the loss due to the viscous dissipation in the interstitial fluid and the dissipation due to inelastic collisions. We use lattice-Boltzmann simulations, to calculate the viscous dissipation as a function of particle volume fraction and Reynolds number (based on the particle diameter and granular temperature). The Reynolds stress induced in the interstitial fluid by the random motion of the particles is also determined. We also consider the case where the interstitial fluid is moving relative to the particles, as would be the case if the particles experienced an external body force.

Uniqueness of BV Entropy Solutions for Quasilinear Parabolic Equations with Arbitrary Degeneracy

PEIDONG LEI, ZHUOQUN WU AND <u>JINGXUE YIN</u>[♯]
Department of Mathematics
Jilin University, Changchun, P. R. China
E-mail: ♯yjx@mail.jlu.edu.cn

In this paper we are concerned with the Cauchy problem for quasilinear parabolic equations of the form

$$\frac{\partial u}{\partial t} = \Delta A(u) + \nabla \vec{B}(u), \quad (x,t) \in Q_T, \tag{1}$$

$$u(x,0) = u_0(x), \quad x \in \mathbb{R}^N, \tag{2}$$

where $Q_T \equiv \mathbb{R}^N \times (0,T)$, $A(s), \vec{B}(s)$ appropriately smooth. The basic assumption is that $A'(s) \geq 0$ which implies that the equation (1) may be arbitrarily degenerate. In other words, the equation is of parabolic and hyperbolic mixed type.

We are much interested in the uniqueness of BV entropy solutions, namely, the solutions with bounded variation satisfying some entropy conditions. First, we use a quite different new approach to define solutions. Roughly speaking, we choose the test functions in the same space, i.e., the space of all functions with bounded variation, as the solutions belong to, and incorporate with the entropy conditions into an integral inequality with an arbitrary test function and an arbitrary test constant. Using this new approach, we are able to separate the reasonable entropy conditions for the solutions. Then, based on the delicate properties of BV functions and BV_x functions, we establish the uniqueness of such solutions.

Time-Asymptotic Nonlinear Waves for Boltzmann Equation

Shih-Hsien Yu
Department of Mathematics
City University of Hong Kong, Hong Kong
E-mail: `mashyu@cityu.edu.hk`

We decompose the Boltzmann equations and its solutions into fluid and non-fluid parts so that the analysis for studying the time-asymptotic nonlinear waves of the compressible Navier-Stokes equations can be ported to study those time-asymptotic nonlinear waves of the Boltzmann equation. The time-asymptotic behavior of shock layers, rarefaction waves, and boundary layers for Boltzmann equations are analyzed.

On the Global Existence of Solution of Prandtl's System

Liqun Zhang
Institute of Mathematics
Academia Sinica, P. R. China
E-mail: `lqzhang@math.ac.cn`

In this paper we establish a global existence of weak solutions to the 2-dimensional Prandtl's system for unsteady boundary layers in the class considered by O. A. Oleinik provided that the pressure is favorable is satisfied. The main results of Oleinik and her co-workers can be summarized as that there exists a unique (for short-time if L is given and fixed, and for arbitrary time if L is small) classical smooth solution to the initial-boundary value problem provided that the initial data satisfy conditions

$$U(x,t) > 0, \quad u_0(x,t) > 0, \quad u_1(y,t) > 0;$$

and

$$\partial_y u_0(x,y) > 0, \quad \partial_y u_1(x,y) > 0.$$

One of the open problems listed at the end of book by Oleinik and Samokhin is: what are the conditions ensuring the global in times existence and uniqueness of solution for arbitrary given L?

The main purpose of this paper is to establish the global (in time) existence of weak solution to the problem for arbitrary finite L and data satisfying

$$U(x,t) > 0, \quad u_0(x,t) > 0,$$

$$u_1(y,t) > 0, \quad v_0(x,t) \leq 0;$$

and

$$\partial_y u_0(x,y) > 0, \quad \partial_y u_1(x,y) > 0.$$

and provided that the pressure is favorable, that is,

$$\partial_x p(t,x) \leq 0.$$

This generalizes the local well-posedness results due to Oleinik. For the proof, we introduce a viscous splitting method so that the asymptotic behavior near the fluid can be estimated more accurately by methods applicable to the degenerate parabolic equations. This is a jointed work with Z.P. Xin.

One-dimensional Compressible Navier-Stokes Equations: The Case When the Far Fields of the Initial Density are in Vacuum

HUIJIANG ZHAO

Wuhan Institute of Physics and Mathematics, The Chinese Academy of Sciences, P. R. China and

IMS and Department of Mathematics, The Chinese University of Hong Kong, Hong Kong

E-mail: hjzhao@wipm.whcnc.ac.cn

This paper is concerned with the Cauchy problem of the one-dimensional compressible Navier-Stokes equations for the case when the far fields of the initial density are in vacuum. Under certain assumptions imposed on the initial data, we show that the corresponding Cauchy problem admits a global smooth solution and some uniqueness results are also obtained. This work is joint with Professor Zhouping Xin.

Local Existence with Minimal Regularity for Semilinear Wave Equations

YI ZHOU

Institute of Mathematics

Fudan University, Shanghai, P. R. China

E-mail: yizhou@fudan.ac.cn

In this talk, we consider the following systems of semilinear wave equations in n space dimensions:

$$\Box\phi = F(\partial\phi) \tag{1}$$

where $\Box = \partial_t^2 - \triangle$ is the wave operator, $\triangle = \sum_{i=1}^n \partial_{x_i}^2$ stands for the Laplacian, $\phi = (\phi^1, \phi^2, \cdots, \phi^m)$, $F = (F^1, F^2, \cdots, F^m)$, $\partial = (\partial_t, \partial_{x_1}, \cdots, \partial_{x_n})$. Moreover, F^i are quadratic functions of $\partial\phi$. We prescribe the following Cauchy data at time $t = 0$:

$$\phi(0, x) = f(x), \quad \partial_t\phi(0, x) = g(x) \tag{2}$$

where $f \in H^s, g \in H^{s-1}$.

We want to determine the minimal value of s such that the Cauchy problem (1)(2) is locally well posed in H^s. The classical local existence theorem requires $s > \frac{n}{2} + 1$ while the scaling limit is $s > \frac{n}{2}$. The correct one turns out to be

$$s > max\{\frac{n}{2}, \frac{n+5}{4}\}. \tag{3}$$

The counter examples which show that (3) can not be improved is due to Lindblad. The positive result is due to Sideris et al (when n=3) and Tartaru (when $n \geq 5$). The proof in the case $n = 2, 4$ was recently supplemented by the author.

when F satisfies the so called "null condition", that is, F^i are all linear combinations of the following null forms:

$$Q_0(\phi, \psi) = \partial_t\phi\partial_t\psi - \sum_{i=1}^n \partial_{x_1}\phi\partial_{x_1}\psi \tag{4}$$

$$Q_{\alpha\beta}(\phi, \psi) = \partial_{x_\alpha}\phi\partial_{x_\beta}\psi - \partial_{x_\beta}\phi\partial_{x_\alpha}\psi \quad (\alpha, \beta = 0, 1, \cdots, n). \tag{5}$$

Then the local well posedness result can be improved. For the first null form, one can prove local well posedness up to the scaling limit $s > \frac{n}{2}$, this is due to Klainerman and Machedon (when n=3,4), Klainerman and Selberg (when n=2), Keel and Tao (when n=1). For the second null form , the correct exponent is

$$s > max\{\frac{n}{2}, \frac{n+3}{4}\}. \tag{6}$$

This is due to Klainerman and Machedon (when n=3,4) and the author (when n=1,2).

Numerical Solutions of Electromagnetic Maxwell Systems

Jun Zou
Department of Mathematics
The Chinese University of Hong Kong, Hong Kong
E-mail: zou@math.cuhk.edu.hk

In this talk we will report some of our recent results about the nodal and edge finite element methods for solving the electromagnetic Maxwell systems in physical domains consisting of more than one media. Due to the jumps in the physical coefficients of the Maxwell equations across the interfaces, electric and magnetic fields may have very weak regularities in the entire domain, and the formulation of the variational problems need some special treatments to allow non-matching grids to be used in regions of different media. Some new frameworks for the convergence analysis of finite element methods and some efficient iterative methods for solving the resulting discrete systems of saddle-point type will be presented. Applications of our results for nonlinear geodynamic problem will be discussed.

This work was supported by Hong Kong RGC Grant CUHK4292/00P.

Abstracts of Contributed Talks

Finite Element Approximation to Evolution Problems in Mixed Form

Daniele Boffi[♯] and Lucia Gastaldi[†]
[♯]*Dipartimento di Matematica, Università di Pavia, Italy*
[†]*Dipartimento di Matematica, Università di Brescia, Italy*
E-mail: [♯]boffi@dimat.unipv.it, [†]gastaldi@ing.unibs.it

This study is a first step towards the analysis of a modification of the immersed boundary method proposed by Peskin.

Mixed finite elements are often used in engineering applications and their analysis has been considered in several papers, starting from the 70's, mainly for the approximation of steady source problems. When a finite element approximation to a mixed is considered, it is well-known that the necessary and sufficient condition for the well-posedness, stability and convergence of the scheme (for any given data) is that two inf-sup constants are bounded below away from zero independently of the meshsize parameter. In the 80's the use of mixed finite elements has been considered also for the approximation of eigenvalue problems and only fairly recently it has been understood that the inf-sup conditions are not the main assumptions in this context.

We consider the finite element approximation of evolution problems in mixed form. There is few mathematical literature on this field, mainly related to the approximation of the heat equation by means of Raviart-Thomas elements, even though mixed finite element schemes have been extensively used for the approximation of evolution problems, in particular in fluid-dynamic applications.

We will give theorems, stating sufficient conditions for the good approximation of the problems under consideration and report on numerical tests confirming our theory. In particular, we present an example of discretization of the heat equation for which the standard inf-sup conditions are satisfied (hence the corresponding steady Poisson problem is well approximated) but which does not provide a good scheme for the evolution heat equation.

Pointwise Error Estimate for an Elliptic System of Quasi-Variational Inequalities

Messaoud Boulbrachene
Department of Mathematics & Statistics
Sultan Qaboos University, Sultanate of Oman
E-mail: boulbrac@squ.edu.om

A number of results on pointwise error estimates for the classical obstacle problem in particular, and variational inequalities (VI) in general, have been achieved in the last three decades, where the most significant ones are due to Nitsche (1977), C. Baiocchi (1977), P. Cortey-Dumont (1985), R.H. Nochetto (1988). However, very little is known on the subject when it comes to quasi-variational inequalities (QVIs) In the present work we investigate the approximation in the L^∞ norm for the following system of QVIs: find $(u^1, ..., u^M) \in (H_0^1(\Omega))^M$ such that

$$\begin{cases} a^i(u^i, v - u^i) \geqq (f^i, v - u^i) \; \forall v \in H_0^1(\Omega), \quad v \leq k + u^{i+1} \\ u^i \leq k + u^{i+1}; \quad \text{with} \quad u^{M+1} \; = \; u^1 \end{cases} \tag{1}$$

where Ω is a bounded smooth domain of R^N, $N \geq 1$, $a^i(.,.)$ are continuous bilinear forms associated with second order elliptic operators, $(.,.)$ denotes the standard inner product in $L^2(\Omega)$, f^i are regular functions, and k is a positive number. The system under consideration plays a key role in solving Hamilton-Jacobi-Bellman equations (HJB) studied by P.L. Lions and J.L Menaldi (1979).

We show that the piecewise linear approximation applied to this system is quasi-optimally accurate in $L^\infty(\Omega)$. Our study includes both the coercive and noncoercive problems, where different approaches are respectively developed and analyzed. As a consequence of our main result, we also derive a pointwise error estimate for the corresponding HJB equations.

The support provided by Sultan Qaboos University (project No Sci/01/02) is gratefully acknowledged.

The Hyperbolic Monge-Ampere Equation: Classical Solutions on the Whole Plane

Yuri Bratkov
Moscow State University, Russia
E-mail: YuriB@vostok.glas.apc.org

On the plane (x, y) the Cauchy problem for the hyperbolic Monge-Ampere equation

$$\begin{cases} A + Bz_{xx} + Cz_{xy} + Dz_{yy} + \operatorname{hess} z = 0 \,, \\ z(0, y) = z^o(y), \quad z_x(0, y) = p^o(y), \quad y \in \mathcal{R} \end{cases}$$

is considered. Here $\operatorname{hess} z = z_{xx}z_{yy} - z_{xy}^2$, A, B, C, D depend on x, y, z, z_x, z_y. The equation is hyperbolic when $C^2 - 4BD + 4A > 0$.

The existence of the C^3-solution on the whole plane is proved. The sufficient conditions are formulated.

Reference

[1]. Yu. N. Bratkov, On the existence of the classical solution of the hyperbolic Monge-Ampere equation on the whole. *Fundamental and Applied Mathematics*, **2** (2000) 379–390. (Russian)

A Family of Partial Differential Equations Arisen from Generalised Markov Branching Processes

Anyue Chen
School of Computing and Mathematical Sciences
University of Greenwich
E-mail: A.Chen@gre.ac.uk

T.B.A.

Perturbation of Semilinear Evolution Equations

Daniel Daners
School of Mathematics and Statistics
University of Sydney, Australia
E-mail: D.Daners@maths.usyd.edu.au

We consider perturbations of semilinear degenerate parabolic evolution equations on Banach spaces. We prove lower semi-continuity of the existence times, and study continuous dependence of solutions with respect to these perturbations. The theory applies to parameter dependent evolution equations and also to boundary value problems on non-smoothly varying domains. We also obtain weak continuity properties of solutions with respect to initial conditions. We will outline some of these applications.

Stationary Euler Equation on Lipschitz Domains in Riemannian Manifolds

Martin Dindoš
Department of Mathematics
Cornell University, USA
E-mail: dindos@math.cornell.edu

The Euler and Navier-Stokes equations are one of the most studied nonlinear equations. They model the flow of an incompressible fluid, (viscous in the case of the Navier-Stokes equation and with zero viscosity in the case of the Euler equation). Both equations give rise to the stationary problems, i.e., when the fluid flows is not time dependent. Stationary problems for both equations are not trivial for example, the existence of a solution for the stationary Navier-Stokes equation on smooth domains in known in dimensions 6 or less.

In our paper we present new results on the existence of stationary solution for the Euler equation in dimensions 4 or less. The main theorem claims that the equation

$$\nabla_u u + dp = f, \quad \delta u = 0, \quad \mathrm{Tr}\ u = g \in B^{2,2}_{1/2}(\partial\Omega)$$

has a solution u in $L^2_1(\Omega, \Lambda^1 TM)$ for any conservative force f and boundary data g such that $\langle g, \nu\rangle = 0$.

Stability of Classes of Solutions to Partial Differential Relations and Quasiconvexity

Alexandre A. Egorov
Sobolev Institute of Mathematics
Siberian Branch of Russian Academy of Sciences, Russia
E-mail: yegorov@math.nsc.ru

We consider classes of solutions $u : U \to \mathbb{R}^m$ to partial differential relations $u^{(l)}(x) \in K$ a.e. in $U \subset \mathbb{R}^n$. Here $u^{(l)}(x)$ stands for the differential of order l of u at $x \in U$ and K is a set in the space $R_s^{mn^l}$ of symmetric l-linear maps from $\mathbb{R}^n$ into $\mathbb{R}^m$. The stability properties of these classes are described in terms of quasiconvex sets. We study stability in the frameworks of the concepts of stability of classes of mappings which were suggested by A.P. Kopylov. These concepts are one general direction of research into the stability phenomena for classes of mappings and involve some well-known results on stability of conformal and isometric transformations established by M.A. Lavrent'ev, P.P. Belinskii, Yu.G. Reshetnyak, F. John, et al. Gradient Young measures are used in our approach to studying stability problems. Part of the results presented in the talk is joint work with M.V. Korobkov.

A Nonlinear Model for Shells with Variable Thickness

Liliana Gratie
Liu Bie Ju Centre for Mathematical Sciences
City University of Hong Kong, Hong Kong
E-mail: mcgratie@cityu.edu.hk

The intensive development of nonlinear shell theory is mainly connected to its wide applicability in thin walled structures, in civil engineering or in aircraft and ships body structures, where lightweight and optimal shape are essential. For these reasons, such "thin" structures usually have non-constant thickness.

In this work, we propose and, using the method of formal asymptotic expansions, we justify a *shell model "of Koiter's type" for nonlinearly elastic shells with variable thickness*, which extends that proposed by Ciarlet [2002] for shells with constant thickness.

We also show that nonlinearly elastic shells with variable thickness have two essentially distinct limit behaviors as their thickness approach zero, either that of a nonlinearly elastic membrane shell or that of a nonlinearly elastic flexural shell with variable thickness. Detailed results and proofs can be found in Gratie [2002].

References

[1]. Ciarlet, P.G. [2002]: A two-dimensional nonlinear shell model of Koiter's type. *Proceedings of the Conference in the Honor of Jean Leray*, Kluwer, Dordrecht, to appear.

[2]. Gratie, L. [2002]: Two-dimensional nonlinear shell model of Koiter's type with variable thickness, to appear.

Improved Accuracy for Locally One-Dimensional Methods for Parabolic Equations Based on Mixed Finite Element Procedures

JIM DOUGLAS, JR.[♯] AND <u>CHIEH-SEN HUANG</u>[†]
[♯]*Department of Mathematics, Purdue University, USA*
[†]*Department of Mathematics, National Sun Yat-Sen University, Taiwan*
E-mail: [†]huangcs@math.nsysu.edu.tw

Classical alternating direction (AD) methods for parabolic equations, based on some standard implicit time stepping procedure such as Crank-Nicolson, can have errors associated with the AD perturbations that are much larger than the errors associated with the underlying time stepping procedure. We show that minor modifications in the AD procedures can virtually eliminate the perturbation errors at an minor additional computational cost. A mixed finite element method is applied in the spactial variables. Similar to the finite difference and finite element methods in spactial variables, we have the same accuracy in time. A convergence analysis can also be shown .

Keywords: Alternating direction method, mixed finite element methods.

Existence and Uniqueness for a Size Structured Population Model with Nonlinear Growth Rate

NOBUYUKI KATO
Department of Mathematics
Shimane University, Japan
E-mail: nkato@math.shimane-u.ac.jp

We give the existence and uniqueness of the solution to a size structured population model with the growth rate depending on the individual's size and its total population at each time. The model comes from the population models of plants in forests or plantations and described by the following initial boundary value problem with nonlocal boundary condition:

$$\text{(NSDP)}\begin{cases} u_t + (V(x,P(t))u)_x = G(u(\cdot,t))(x), \quad x \in [0,l),\ 0 \le t \le T, \\ V(0,P(t))u(0,t) = C(t) + F(u(\cdot,t)), \quad 0 \le t \le T, \\ u(x,0) = u_0(x), \quad x \in [0,l). \\ P(t) = \int_0^l u(x,t)dx \end{cases} \tag{1}$$

The unknown function $u(x,t)$ stands for the density of population of size x at time t and $P(t) = \int_0^l u(x,t)dx$ represents the total population, where $0 < l \leq \infty$ is the maximum size. The function $V(x, P(t))$ is the nonlinear growth rate function depending on the size x and the total population $P(t)$ at each time t. The functions F and G correspond to the birth and aging functions respectively and the function C represents the inflow of zero-size individuals (i.e. newborns) in the population from an external source (for example, seeds carried by the wind or placed in a plantation).

Our results are the generalizations of the ones due to A. Calsina and J. Saldaña in 1995, where they treated the Gurtin-MacCamy type model, i.e., F and G have the following forms:

$$F(u(\cdot,t)) = \int_0^l \beta(x, P(t))u(x,t)dx, \quad G(u(\cdot,t))(x) = -m(x, P(t))u(x,t). \tag{2}$$

Their arguments strictly depend on these special forms. For a general model as above, we need a different method. We emphasize that for the uniqueness, some additional assumptions are needed compared with the existence result and it is not known whether the uniqueness is obtained under the same conditions as in the existence result.

Optimal Sobolev Imbeddings Involving Rearrangement-Invariant Norms

Ron Kerman
Department of Mathematics
Brock University, Canada
E-mail: rkerman@spartan.ac.BrockU.CA

Let m and n be positive integers with $n > 1$ and $m < n$. We consider Sobolev imbedding inequalities that estimate the size (as measured by a rearrangement-invariant norm) of a smooth function u, compactly supported in the unit ball of n-dimensional Euclidean space, by the size (as measured by another such norm) of the total m-th order gradient of u. Attention is focused on when such inequalities are optimal. Our results yield best possible refinements of such (limiting) Sobolev inequalities as those of Trudinger and Brezis-Wainger.

This is joint work with Lubos Pick.

Mathematical Analysis of the Quasilinear Effects in a Hyperbolic Model of Blood Flow Through Compliant Axi-symmetric Vessels

Eun Heui Kim
Department of Mathematics
California State University, USA
E-mail: ekim4@csulb.edu

This is a joint work with S. Canic. We present a mathematical analysis of the quasilinear effects arising in a hyperbolic system of partial differential equations modeling blood flow through large compliant vessels. The equations are derived using asymptotic reduction of the incompressible Navier-Stokes equations in narrow, long channels. We discuss the estimates on the initial and boundary data which imply strict hyperbolicity in the region of smooth flow and outline a proof of a general theorem which provides conditions under which an initial-boundary value problem for a quasilinear hyperbolic system admits a smooth solution. Using this result we show that pulsatile flow boundary data always give rise to shock formation (high gradients in the velocity and inner vessel radius). We estimate the time and the location of the first shock formation and show that in a healthy individual, shocks form well outside the physiologically interesting region (2.8 meters downstream from the inlet boundary). We also present numerical results on the model problem.

Symmetry Breaking Phenomena in an Optimization Problem for Some Nonlinear Elliptic Equation

KAZUHIRO KURATA
Department of Mathematics
Tokyo Metropolitan University, Japan
E-mail: kurata@comp.metro-u.ac.jp

In this talk, we consider an optimization problem associated with the following nonlinear elliptic boundary value problem:

$$-\Delta u + \lambda u^p = \chi_D, u > 0, \quad (x \in \Omega), \qquad u = 0 \quad (x \in \partial\Omega),$$

where Ω is a bounded domain in $\mathbf{R}^n$ with smooth boundary, $\lambda > 0$, and $1 \leq p < (n+2)/(n-2)$ if $n \geq 3$ and $1 \leq p < +\infty$ if $n = 1, 2$. Here, D is a measurable subset of Ω which belongs to the class: $\mathcal{C}_\beta = \{D \subset \Omega \quad | \quad |D| = \beta\}$ for the prescribed $\beta \in (0, |\Omega|)$. It is well-known that for any $D \in \mathcal{C}_\beta$, there exists a unique solution $u \in H_0^1(\Omega)$, we denote it by u_D, to the nonlinear boundary value problem above and the solution u_D is obtained as a global minimizer of the functional:

$$J_D(v) = \frac{1}{2}\int_\Omega |\nabla v|^2 \, dx + \frac{\lambda}{p+1}|v|^{p+1} \, dx - \int_\Omega \chi_D v \, dx$$

on $H_0^1(\Omega)$. Now, we consider the optimization problem: $E_{\beta,\Omega} = \inf_{D \in \mathcal{C}_\beta} J_D(u_D)$. If $E_{\beta,\Omega}$ is attained by $D^* \in \mathcal{C}_\beta$, then we call D^* an optimal configuration. In this talk, we present our recent results on the existence, uniqueness and non-uniqueness, symmetry preserving and symmetry breaking phenomena of the optimal configuration D^* to this optimization problem in various settings. When $\lambda = 0$, similar results has been obtained by Cuccu and Porru in [1]. We extend their ideas to be adapted to our setting and overcome some difficulty to show symmetry breaking in the nonlinear case. We should also mention that such symmetry breaking phenomena was first discovered in the optimization of the first Dirichlet eigenvalue to the Schrödinger operator in [2].

References

[1]. F. Cuccu, G. Porru, Optimization in a problem of heat conduction, preprint 2001.

[2]. S. Chanillo, D. Griser, M. Imai, K. Kurata, I. Ohnishi, Symmetry breaking and other phenomena in the optimization of eigenvalues for composite membranes. *Comm. Math. Phys.*, **214** (2000), 315–337.

Stable Positive Bifurcating Equilibria for a System of Damped Wave Equations

Anthony W. Leung
Department of Mathematical Sciences
University of Cincinnati, USA
E-mail: Anthony.Leung@uc.edu

A system of nonlinear damped wave equations with symmetric linear part is investigated. A positive steady-state bifurcates from the trivial solution as a parameter changes. The spectrum of the linearized operator is studied. Then the stability of the positive steady-state is considered as a solution of the nonlinear hyperbolic system. Asymptotic stability results are found for the solutions in bounded domains of dimensions larger than or equal to one. Bifurcation methods are used to prove the existence of positive steady-states, and semigroup methods are used to study stability. Stability results are obtained although the semigroup is not analytic. In applications, the interacting waves can be electromagnetic fields, lasers or various forms of vibrations.

A Fourth Order Equation Modeling Beams on Elastic Bearings

To Fu Ma
Departamento de Matemática
Universidade Estadual de Maringá, Brazil
E-mail: matofu@dma.uem.br

In this talk we consider the global existence for the fourth order equation

$$u_{tt} + u_{xxxx} - M(\int_0^L |u_x(x,t)|^2 dx)u_{xx} = 0,$$

with nonlinear boundary conditions modeling beams on elastic foundations. The boundary stabilization for a related transmission problem involving a system of two Euler-Bernoulli equations is also considered.

References

[1]. E. Feireisl, Non-zero time periodic solutions to an equation of Petrovsky type with nonlinear boundary conditions: slow oscillations of beams on elastic bearings. *Annali della Scuola Normale Superiore di Pisa*, **XX** (1993) 133–146.

[2]. M. R. Grossinho and S. Tersian, The dual variational principle and equilibria for a beam resting on a discontinuous nonlinear elastic foundation. *Nonlinear Analysis*, **41** (2000) 417–431.

[3]. T. F. Ma, Existence results for a model of nonlinear beam on elastic bearings. *Applied Mathematics Letters*, **13** (2000) 11–15.

[4]. T. F. Ma, Boundary stabilization for a non-linear beam on elastic bearings. *Mathematical Methods in the Applied Science*, **24** (2001) 583–594.

Analysis of Coupled Quantum-Classical Models for the Electron Transport in Nanostructures

FLORIAN MÉHATS
Laboratoire MIP
Université Paul Sabatier, France
E-mail: mehats@mip.ups-tlse.fr

This is a joint work with N. Ben Abdallah and G. Quinio.

We are interested in the modelling of the transport and interaction of electrons partially confined in a nanostructure. The particles are assumed to have a wave behaviour in the confinement directions (z) and to behave like point particles in the directions parallel to the electron gas (x). This leads to a coupling between Quantum and Classical models in the momentum variable.

For each fixed x and at each time t, the eigenfunctions χ_p and the eigen-energies ϵ_p of the Schrödinger operator (2) in the z are computed. The occupation number of each eigenfunction is calculated through the resolution of a Vlasov equation (1) in the x direction, the force field being the gradient of the eigen-energy. The whole system is coupled to the Poisson (3) equation for the electrostatic interaction. In the case of a bounded domain, the whole system writes

$$\partial_t f_p + v \cdot \nabla_x f_p - \nabla_x \epsilon_p \cdot \nabla_v f_p = 0, \tag{1}$$

$$\begin{cases} -\dfrac{1}{2}\,\partial_{zz}\chi_p + V\chi_p = \epsilon_p \chi_p\,, \\ \chi_p(t,x,\cdot) \in H_0^1(0,1), \qquad \displaystyle\int_0^1 \chi_p\,\chi_q\,dz = \delta_{pq}\,, \end{cases} \tag{2}$$

$$-\Delta V = \sum_{p\geq 1} \left(\int_{\mathbb{R}^2} f_p\,dv \right) |\chi_p|^2\,, \tag{3}$$

with Cauchy data $f_p(0,x) = f_p^0$ and suitable boundary conditions.

In a first part, existence of weak solutions will be shown for boundary value problems in the stationary and time dependent regimes. The proofs rely on the one hand on the study of quasistatic Schrödinger-Poisson systems and on the other hand on an energy estimate (for the time dependent case) and on supersolution techniques (for the stationary case).

Then the adaptation to the case of the whole space will be presented. The existence of a unique classical solution is shown, the proof being based on an iterative scheme.

Dynamics of a Parametrically Excited PDE: A Perturbation Approach

M. Mofid[♯] and M. R. Shadnam[†]
[♯] *Civil Engineering Department, Sharif University of Technology, Tehran, Iran*
[†] *Civil Engineering Department, University of Alberta, Canada*
E-mail: [♯]mofid@sharif.edu, [†]mshadnam@ualberta.ca

The dynamics of plates under influence of relatively large masses, moving periodically on the plate is considered. The time-dependent equations of motion in are solved by perturbation method. Instability regions of parametric resonances are identified and the moving mass effect is shown to significantly affect the transient response of the beam. Importance of modal interaction arising out of the possible internal resonance is highlighted. The method presented in this investigation is general and can be applied to general moving mass and moving force systems as well.

The dynamic behavior of structures under influence of moving loads is a subject of considerable importance, and much attention had been given to the corresponding mathematical problem. The fundamental mathematical complexity encountered in this problem lies in the fact that one of the coefficients of operator describing the motion is a function of space and time. This is caused by the presence of a Dirac-delta function as a coefficient necessary for a proper description of motion.

The used moving mass model renders the structural system a parametrically excited one and hence causes instability [1].

Reference

[1]. V. V. Bolotin. *The dynamic stability of elastic systems*, Holden Day, San Francisco (1964).

Solutions with Internal Jump for an Autonomous Elliptic System of Bistable Type

Carolus J. Reinecke[♯] and Guido Sweers[†]
[♯] *Department of Mathematics, University of Potchefstroom, South Africa*
[†] *TU delft, The Netherlands*
E-mail: [♯]WSKCJR@puknet.puk.ac.za

We consider the following system of semilinear elliptic equations

$$\left\{\begin{array}{rcll} -\varepsilon^2\Delta u &=& f(u)-v & \text{in} \quad \Omega; \\ \gamma v-\Delta v &=& \delta u & \text{in} \quad \Omega; \\ u &=& v=0 & \text{on} \quad \partial\Omega. \end{array}\right. \tag{1}$$

We assume Ω to be a smooth bounded domain in $\mathbb{R}^N$, with $N \geq 1$, while γ is larger than the first eigenvalue of $-\Delta$ on Ω subjected to homogeneous Dirichlet boundary conditions.

We take $\varepsilon > 0$ and $\delta \geq 0$ as parameters. The nonlinearity we assume for simplicity to be $f(u) = u(u-1)(a-u)$ with $0 < a < 1/2$, although other more general nonlinearities can also be treated. We observe that the system is coupled in a noncooperative way, and hence is not order preserving. This leads to a richer solution structure. In particular for small $\delta \geq 0$ the solutions to (1) are similar to the solutions to the scalar equation

$$\begin{cases} -\varepsilon^2 \Delta u &= f(u) \quad \text{in} \quad \Omega; \\ \quad\quad u &= 0 \quad\quad \text{on} \quad \partial\Omega, \end{cases} \tag{2}$$

for which it is known that, under certain assumptions on Ω and for ε small, there exist only two nontrivial solutions. We shall present results which show how this simple solution structure becomes more complex as δ increases.

On an Inequality by N. Trudinger and J. Moser

Bernhard Ruf
Dipartimento di Matematica
Università di Milano, Italy
E-mail: ruf@mat.unimi.it

It was shown by N. Trudinger and J. Moser that for functions in $H_0^1(\Omega)$ with $\int_\Omega |\nabla u|^2 dx \leq 1$ (where Ω is a bounded domain in $\mathbb{R}^2$) the integral $\int_\Omega \exp(\alpha u^2) dx$ remains uniformly bounded by some constant $C_\alpha = C_\alpha(\Omega)$, for $\alpha \leq 4\pi$, and becomes infinite for $\alpha > 4\pi$. L. Carleson and A. Chang proved that for the limiting case $\alpha = 4\pi$ there exists a corresponding extremal function, in the case that Ω is the unit ball in $\mathbb{R}^2$. For proving this they introduced the maximal limit of all "concentrating" sequences of normalized functions; let us call this limit the "Carleson-Chang limit".

We give a new proof, a generalization, and a new interpretation of this result. In particular, we give an explicit sequence which converges to the Carleson-Chang limit.

The constant $C_{4\pi}(\Omega)$ mentioned above becomes infinite for unbounded domains. We show that there exists a uniform bound, independent of Ω, if the Dirichlet norm is replaced by the standard H_0^1-norm $\|u\|^2 = \int_\Omega |\nabla u|^2 + |u|^2 dx$. Furthermore, an explicit formula for the corresponding Carleson-Chang limit is given.

Pressure-correction Algebraic Fractional-step Schemes for the Unsteady Navier-Stokes Incompressible Equations

Fausto Saleri[♯] and Alessandro Veneziani
Dipartimento di Matematica "F. Brioschi"
Politecnico di Milano, Italy
E-mail: [♯]fausal@mate.polimi.it

One of the most successful approaches to solve the Navier-Stokes equations for incompressible flows is provided by the class of the projection methods at the differential and, more recently, at the algebraic level.

In this talk we present a new family of algebraic projection methods based on inexact LU factorization of the full discretized system. In particular, we will introduce a pressure-correction scheme that resembles a well known projection scheme based on a differential approach (see [3], [2]). We will illustrate the effect of the LU inexact factorization when used as a solver for the fully discretized Navier-Stokes problem. We will refer to a finite element discretization in space and a finite difference discretization in time.

Moreover, we consider the same approach as a preconditioner for the same problem. In this respect, we will build a new preconditioner which in some sense generalize the well known Caouet-Chabard preconditioner [1] and which seems to be well suited not only for the generalized Stokes problem. This is confirmed by several numerical results.

References

[1]. J. Caouet and J.-P. Chabard, Some fast 3d finite element solvers for the generalized Stokes problem. *Int. Journ. for Numer. Methods in Fluids*, **8** (1988), 869–895.

[2]. A. Prohl. *Projection and Quasi-Compressibility Methods for Solving the Incompressible Navier-Stokes Equations*, Springer-Verlag, 1997.

[3]. L. Timmermamns, P. Minev, and F.V. De Vosse, An approximate projection scheme for incompressible flows using spectral methods. *Int. Journ. for Numer. Methods in Fluids*, **22** (1996), 673–688.

Removable Singularities and Quasilinear Parabolic Equations with an Exponential Term

Luis M. R. Saraiva
Mathematics Department, Faculty of Sciences
Lisbon University, Portugal
E-mail: mmff5@lmc.fc.ul.pt

Saraiva (1985) studied the characterization of sets of weighted capacity zero, and used it for obtaining removable singularities results for weak solutions of degenerate quasilinear parabolic equations. In this paper we aim at obtaining a removable singularities result quasilinear parabolic equations with an exponential term, using the above mentioned characterization, for a particular case where an embedding into an Orlicz space holds. We will study the equation

$$u_t = \mathrm{div} A(x,t,u,u_x) + B(x,t,u,u_x) + e^{bu},$$

where A and B are respectively vector and scalar valued measurable functions defined on $\Omega \times \mathbb{R} \times \mathbb{R}^N$, Ω an open set in $\mathbb{R}^{N+1}$, and satisfying certain structure inequalities. We will adapt the technique used by Saraiva (1985) to this particular case and use the embedding into an Orlicz space to compensate the existence of the exponential term.

References

[1]. R. A. Adams. *Sobolev Spaces*. New York: Academic Press, 1975.

[2]. O. V. Besov, V. P. Il'in and S. M. Nikols'ki. *Integral Representation of Functions and Imbedding Theorems.* **Vol.1** New York: John Wiley, 1978.

[3]. D. E. Edmunds and W. D. Evans. *Spectral Theory and Differential Operators.* Oxford: Clarendon Press, 1987.

[4]. L. M. R. Saraiva, Removable singularities and quasilinear parabolic equations. *Proc. London Math. Soc.*, (3) **48** (1984), 385–400.

[5]. L. M. R . Saraiva, Removable singularities of solutions of degenerate quasilinear equations. *Ann. Mat. Pura Appl.*, (IV) CXLI, (1985), 187–221.

[6]. L. M. R. Saraiva, Singular sets of anisotropic weighted capacity zero and degenerate quasilinear equations. *Proc. Royal Soc. Edinburgh*, **126 A** (1996), 209–224.

An Inverse Nonlinear Diffusion Problem

ABDULLAH SHIDFAR[♯] AND KAVOOS TAVAKOLI[†]
Department of Mathematics
Iran University of Science and Technology, Iran
E-mail: [♯]shidfar@iust.ac.ir, [†]mathktavakoly@yahoo.com

The problem of identifying of the coefficient in a square porous medium will be considered. It is shown that under certain conditions of data f, g, and for a properly specified class A of admissible coefficients, there exists at least one a_* in admissible coefficients such that (a_*, u) is a solution of the corresponding inverse problem.

Connections Between the Convective Diffusion Equation and the Forced Burgers Equation

NEJIB SMAOUI AND FETHI BELGACEM
Department of Mathematics and Computer Science
Kuwait University, Kuwait
E-mail: {smaoui, belgacem}@mcs.sci.kuniv.edu.kw

The convective diffusion equation with drift $b(x)$ and indefinite weight $r(x)$:

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial x}\left[a\frac{\partial \phi}{\partial x} - b(x)\phi\right] + \lambda r(x)\phi, \tag{1}$$

is introduced as a model for population dispersal. Strong connections between Eq.(1) and the forced Burgers equation with positive frequency $(m \geq 0)$:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - u\frac{\partial u}{\partial x} + mu + k(x), \tag{2}$$

are established through the Hopf-Cole transformation. Eq.(2) is a prime prototype of the large class of quasilinear parabolic equations given by

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial (f(v))}{\partial x} + g(v) + h(x). \tag{3}$$

A compact attractor and an inertial manifold for the forced Burgers equation are shown to exist via the Kwak transformation. Consequently, existence of an inertial manifold for the convective diffusion equation is guaranteed. Eq.(2) can be interpreted as the velocity field precursed by Eq.(1). Therefore, the dynamics exhibited by the population density in Eq.(1) show their effects on the velocity expressed in Eq.(2). Numerical results illustrating some aspects of the previous connections are obtained by using a pseudospectral method.

Semilinear Elliptic Equations in Unbounded Domain

Robert Stańczy
Faculty of Mathematics
University of Łódź, Poland
E-mail: stanczr@math.uni.lodz.pl

The existence of a bounded, radial solution in unbounded domain $\Omega = \mathbb{R}^n \setminus B(0,1)$ with $n \geq 3$ of the nonlinear elliptic problem

$$\begin{aligned} &\Delta u = f(x,u) \ \text{ in } \ \Omega, \\ &u = 0 \ \text{ on } \ \partial\Omega \end{aligned} \tag{1}$$

is proved under some asymptotic and sign condition on f. Since the above problem is at resonance (one has one-dimensional kernel spanned by $u_0(x) = 1 - |x|^{2-n}$) we perturb the linear operator to get $\Delta - \lambda I$ for $\lambda > 0$ thus reducing perturbed equation to the operator equation $u = (\Delta - \lambda I)^{-1} Nu$ in the space of bounded and continuous function (where N stands for Nemytski operator generated by f). To this one can apply any suitable fixed point theorem. Having obtained solution of the perturbed equation we show the existence of some sequence u_{λ_n} convergent to the Hölder continuous solution of the resonant problem (assuming f to be Hölder continuous with respect to x). In the case of nonlinearity f depending radially on $|x|$ we prove analogous existence result, reducing equation (1) to BVP for ODE on half-line. Under stronger sign assumption existence of a positive solution is also obtained.

Existence of Renormalized Solutions for Reaction Diffusion Equations

Satoru Takagi
Department of Mathematical Sciences
Waseda University, Japan
E-mail: satoru@toki.waseda.jp

We introduce the notion of renormalized solutions and consider existence of such solutions for a reaction diffusion equation which has extremely large initial data.

Let Ω be a bounded domain in $\mathbb{R}^N$, $N \geq 1$, with a Lipschitz boundary $\partial\Omega$ whenever $N \geq 2$, and let $T > 0$. We consider the following reaction diffusion equation

$$\text{(RD)} \quad \begin{cases} \dfrac{\partial u}{\partial t} - \triangle u = u^2 & \text{in } Q = (0,T] \times \Omega, \\ u = 0 & \text{on } \Sigma = [0,T] \times \partial\Omega, \\ u(0) = g & \text{in } \Omega, \end{cases}$$

where $g : \Omega \to \mathbb{R}$ is a given nonnegative and nontrivial function.

Reaction diffusion equations are related to many equations arising from mathematical biology, however, we have not yet obtained global existence results for reaction diffusion equations in the case of extremely large initial data in an appropriate sense. When $1 \leq N \leq 2$, in general, the local solution of (RD) blows up in finite time while the initial value g is nontrivial, and when $N > 2$, if g is small then (RD) has a global solution decaying to 0 as $t \to \infty$. On the other hand, if g is large enough then the local solution blows up in finite time. In the case of large initial data, it is a good way to utilize the concept of renormalized solutions which are defined by truncated functions. We there introduce the notion of renormalized solutions and prove existence of such solutions for the reaction diffusion equation (RD) which has extremely large initial data g.

References

[1]. S. Takagi, Existence of renormalized solutions for reaction diffusion equations, submitted.

[2]. X. Wang, On the Cauchy problem for reaction-diffusion equations. *Trans. Amer. Math. Soc.*, **337** (1993) 549–590.

An Inverse Problem of Heat Equation

ABDULLAH SHIDFAR$^\sharp$ AND KAVOOS TAVAKOLI†
Department of Mathematics
Iran University of Science and Technology, Iran
E-mail: $^\sharp$shidfar@iust.ac.ir, †mathktavakoly@yahoo.com

---

In this paper we consider an inverse problem of linear heat equation with nonlinear boundary condition. We identify the temperature and the unknown radiation term form an overspecified condition on the boundary.

Finite Element Approximation of Conformal Mappings and Its Applications to a Free Boundary Problem

TAKASHI SUZUKI[♯] AND TAKUYA TSUCHIYA[†]
[♯] *Graduate School of Engineering Science, Osaka University, Japan*
[†] *Department of Mathematical Sciences, Ehime University, Japan*
E-mail: [♯]suzuki@sigmath.es.osaka-u.ac.jp,
[†]tsuchiya@math.sci.ehime-u.ac.jp

In this lecture we discuss finite element approximation of conformal mappings. Then we apply the obtained scheme to compute solutions of a classical free boundary problem — the **dam** (or filtration) **problem**. Let $B \subset \mathbb{R}^2$ be the unit disk and $\gamma \subset \mathbb{R}^2$ a given closed Jordan curve. Conformal mappings $x(u, v)$, $((u, v) \in B)$ are defined on B as solutions to the following partial differential equations:

$$\Delta x = 0 \quad \text{in } B,$$

$$|x_u|^2 - |x_v|^2 = (x_u, x_v) = 0 \quad \text{in } B,$$

$$x(\partial B) = \gamma \text{ and } x|_{\partial B} : \partial B \to \gamma \text{ is homeomorphic.}$$

Here, x_u, x_v are the partial derivatives with respect to u, v, respectively, and $(\cdot, \cdot)$, $|\cdot|$ are the inner product in $\mathbb{R}^2$ and the 2-norm defined by that.

For the conformal mappings, the following variational principle has been known. Let the subset X_γ of mappings on B be defined by

$$X_\gamma := \left\{\psi \in C(\overline{B}; \mathbb{R}^2) \cap H(B; \mathbb{R}^2) \;\middle|\; \psi(\partial B) = \gamma,\ \psi|_{\partial B} \text{ is monotone}\right\},$$

where $\psi|_{\partial B}$ being monotone means that $(\psi|_{\partial B})^{-1}(p)$ is connected for all $p \in \gamma$. Then, a map $x \in X_\gamma$ is a solution of the above system if and only if $x \in X_\gamma$ is a stationary point of the energy functional $D(x)$ in X_γ. Here, D is the Dirichlet integral defined by

$$D(\psi) := \int_B |\nabla \psi|^2 \mathrm{d}u \mathrm{d}v.$$

To find a conformal mapping defined on B, therefore, we just need to find a stationary point of the Dirichlet integral in X_γ.

Here, we approximate the Dirichlet integral by piecewise linear finite elements. It has been shown that, if the Jordan curve is rectifiable, the finite element conformal mappings converge to the exact conformal mappings when triangulation is getting finer.

We then try to apply this scheme to solve the dam (or filtration) problem [1], [2], a typical and classical free boundary problem. A detail of formulation and numerical examples will be given at the lecture.

References

[1]. A. Friedman. *Variational Principles and Free-Boundary Problems*, Wiley, 1982.

[2]. D. Kinderlehrer, G. Stampacchia. *An Introduction to Variational Inequalities and their Applications*, Academic, 1980.

Paley-Wiener Theorem for Dunkl Transform

Vu Kim Tuan
Department of Mathematics and Computer Science
Kuwait University, Kuwait
E-mail: vu@mcs.sci.kuniv.edu.kw

For $\alpha \in R^n \backslash \{0\}$, let σ_α be the reflection in the hyperplane $H_\alpha \subset R^n$ orthogonal to α. A finite set $\mathcal{R} \subset R^n \backslash \{0\}$ is called a root system if $\mathcal{R} \cap R \,.\, \alpha = \{\pm\alpha\}$ and $\sigma_\alpha \mathcal{R} = \mathcal{R}$ for all $\alpha \in \mathcal{R}$. For a given root system $\mathcal{R}$ the reflections σ_α generate a finite group $W \subset O(n)$. All reflections in W correspond to suitable pairs of roots. For a given $\beta \in R^n \backslash \cup_{\alpha \in \mathcal{R}} H_\alpha$, we fix the positive subsystem $\mathcal{R}_+ = \{\alpha \in \mathcal{R} : (\alpha, \beta) > 0\}$. We assume the root system $\mathcal{R}$ is normalized in the sense that $|\alpha| = \sqrt{2}$ for all $\alpha \in \mathcal{R}$. A function $k : \mathcal{R} \to C$ on a root system $\mathcal{R}$ is called a multiplicity function if it is invariant under the action of the associated reflection group W. Denotes the number of conjugacy classes of reflections by m. Let $K = C^m$.

The Dunkl operators $T_\zeta, \zeta \in R^n$, on R^n associated with the finite reflection group W and multiplicity function k are given by

$$T_\zeta f(x) = \partial_\zeta f(x) + \sum_{\alpha \in \mathcal{R}_+} k(\alpha)\alpha_i \,.\, \frac{f(x) - f(\sigma_\alpha)}{< \alpha, x >}.$$

Consider the system

$$T_\zeta(k) f = (\lambda, \zeta) f. \tag{1}$$

There exists an open set $K^{reg} \subset K$ invariant under complex conjugation and containing $\{k \in K | \Re(k) \geq 0\}$ such that the solution space of (1) is 1-dimensional for all $k \in K^{reg}$ and $\lambda \in C^n$. This solution space contains a unique function $Exp_G(\lambda, k, .)$ such that $Exp_G(\lambda, k, 0) = 1$.

Let $\Re(k) \geq 0$. Put

$$w_k(x) = \prod_{\alpha \in \mathcal{R}_+} |(\alpha, x)|^{2k_\alpha}.$$

The Dunkl transforms are defined as follows

$$(D_k f)(\lambda) = \frac{1}{c_k} \int_{R^n} f(x) Exp_G(-i\lambda, k, x) w_k(x) dx,$$

$$(E_k f)(x) = \frac{1}{c_k} \int_{R^n} f(\lambda) Exp_G(i\lambda, k, x) w_k(\lambda) d\lambda.$$

The constant c_k is known as a Mehta-type integral. The Plancherel theorem for the Dunkl transforms says that D_k and E_k are unitary operators on $L_2(R^n, |w_k(x)|\, dx)$, and they are the inverses of each other.

In this talk we establish a Paley-Wiener-type theorem for the Dunkl transforms of functions with compact support. The characterization is formulated on R^n without passing to complexification.

Bifurcation Problem for a Semilinear Elliptic Equation Arising in Population Dynamics, Having Nonlinear Boundary Conditions

KENICHIRO UMEZU
Faculty of Engineering
Maebashi Institute of Technology, Japan
E-mail: ken@maebashi-it.ac.jp

In this talk we discuss a bifurcation problem of positive solutions for the following semilinear elliptic boundary value problem with a nonlinear boundary condition.

$$-\Delta u = \lambda(m(x) - au)u \text{ in } D, \qquad \frac{\partial u}{\partial \mathbf{n}} = b(x)g(u) \text{ on } \partial D,$$

where $D \subset \mathbf{R}^N$, $N \geq 2$, is a bounded domain with smooth boundary ∂D, $\lambda > 0$ is a parameter, $m \in C^\theta(\overline{D})$, $0 < \theta < 1$, may change its sign, $a > 0$ is a constant, $b \in C^{1+\theta}(\partial D)$ satisfying $b \geq 0$, $g \in C^{1+\theta}([0, \delta])$ for any $\delta > 0$ such that $g(0) = 0$, and $\mathbf{n}$ is the unit outer normal to ∂D.

This talk is devoted to the study of the existence and qualitative behavior of positive solutions when $\lambda \to +0$. In particular, we investigate *bifurcation to the right* emanating from the origin $(\lambda, u) = (0, 0)$, meaning that there exist positive solutions $(\lambda_j, u_j) \in (0, \infty) \times C^{2+\theta}(\overline{D})$ satisfying $(\lambda_j, u_j) \to 0$. In the homogeneous Neumann case $\partial u / \partial \mathbf{n} = 0$, it is known that $(\lambda, u) = (0, 0)$ is a bifurcation point to the right if, and only if, $\int_D m dx = 0$. Our main result generalizes this result to the nonlinear case. The global structure of bifurcation curves is also considered.

The Hermite-quadratic and Hermite-cubic Finite-element Approximations to the Non-linear Problems

IRENEUSZ WINNICKI
Faculty of Civil Engineering, Chemistry and Technical Physics
Military University of Technology (MUT), Warsaw, Poland
E-mail: ireneusz_winnicki@poczta.onet.pl

For the inviscid Burger's equation written in the flux form the difference schemes in Hermitian finite-element spaces $V^{(2)}$ and $V^{(3)}$ are constructed. Several examples of solutions are included. Let us consider the equation: $\frac{\partial \phi}{\partial t} + \frac{1}{2}\frac{\partial \phi^2}{\partial x} = 0$. Applying the finite-difference method, it is usually approximated with the differential-difference equation: $\frac{d\phi_i}{dt} + \frac{1}{2}\left(\frac{\phi_{i+1}^2 - \phi_{i-1}^2}{2\Delta x}\right) = 0$. Using the finite-element method in Lagrangian space with *chapeau* expansion functions $\varphi_i(x)$ one can obtain more complicated non-linear difference scheme: $\frac{d}{dt}\left(\frac{\phi_{i-1} + 4\phi_i + \phi_{i+1}}{6}\right) + \frac{1}{\Delta x}\left[-\left(\phi_i^s + 2\phi_{i-1}^s\right)\phi_{i-1}^{s+1} - \left(\phi_{i-1}^s - \phi_{i+1}^s\right)\phi_i^{s+1} + \left(\phi_i^s + 2\phi_{i+1}^s\right)\phi_{i+1}^{s+1}\right] = 0$, where $s-$ the number of iteration on the upper time level. The problem will increase, if we use the Hermite-quadratic or Hermite-cubic finite-element approximations

$$\phi(x,t) = \sum_i \left[\phi_i^f(t)\psi_i^f(x) + \phi_i^d(t)\psi_i^d(x)\right] \quad \psi_i^f(x), \psi_i^d(x) \text{ - quadratic or cubic functions}$$

The finite-element schemes for the term $\frac{1}{2}\frac{\partial \phi^2}{\partial x}$ have very complicated forms. They are solved iteratively. They will be discussed during the presentation.

Remark: This paper is sponsored by the Committee for Scientific Research, Warsaw, Poland.

A Modified Nonlinear Galerkin Method for Solving Ginzburg-Landau Equation

Yujiang Wu
Department of Mathematics
Lanzhou University, P. R. China
E-mail: myjaw@lzu.edu.cn

For solving the Ginzburg-Landau equation $u_t - \nu u_{xx} + \kappa u^3 - \gamma u = f,\ u(0,x) = u_0(x)$, we introduce a kind of modified nonlinear Galerkin method as follows:

$$\frac{dy_m}{dt} + \nu A y_m + P_m\{\kappa(y_m^3 + 3y_m z_m^2) - \gamma y_m\} = P_m f,$$

$$\nu A z_m + z_m^{(2)} + (P_{2m} - P_m)\{\kappa(y_m + z_m^{(1)})^3 - \gamma(y_m + z_m^{(1)})\} = (P_{2m} - P_m)f,$$

$$\nu A z_m^{(2)} + (P_{2m} - P_m)\{3\kappa(y_m + z_m^{(1)})^2 - \gamma)[P_m f - A y_m - P_m(\kappa y_m^3 - \gamma y_m)]\} = 0$$

$$\nu A z_m^{(1)} + (P_{2m} - P_m)(\kappa y_m^3 - \gamma y_m) = (P_{2m} - P_m)f,$$

$$y_m(0) = P_m u_0$$

The convergence of the modified method is analyzed and some numerical examples are shown in the article.

Remark: This project is supported by Foundation for University Key Teacher by the Ministry of Education grant GG-110-73001-1014.

Keywords: nonlinear Galerkin methods, Ginzburg-Landau equation, convergence.

Asymptotic Analysis of Dynamic Problems for Elastic Shells

Li-ming Xiao
Department of Computer Science
Guangdong Normal Polytechnic, P. R. China
E-mail: xlmwhj@21.com

By the method of asymptotic analysis we have got the three dynamic elastic shell models. First, we consider a family of linearly elastic Shells, clamped along their entire lateral face, all having the same middle surface. We make an essential geometrical assumption on the middle surface that the middle surface of shells is "Uniformly Elliptic", the mapping of middle surface and the boundary for its definite domain are smooth enough. If the applied body force density is of order 0 with respect to the half thickness of shell, starting from the three-dimensional dynamic equations of linear elastic shell we get the two-dimensional dynamic equations of membrane shell by letting the thickness of shell go to zero.

Second, we consider a family of linearly elastic shells, all having the same middle surface. The shells are clamped on a portion of their lateral face, we make an essential geometrical assumption that the space of inextensional displacements contains non-zero functions. If the applied body force density is of order 2 with respect to the half thickness of shell, starting from the three-dimensional dynamic equations of linear elastic shell we get the two-dimensional dynamic equations of flexural shell by letting the thickness of shell go to zero.

Lastly, starting from the two-dimensional dynamic equations of Koiter's shell we also get the two-dimensional dynamic equations of membrane shell and the two-dimensional dynamic equations of flexural shell by letting the thickness of shell go to zero, this gives the justification for the two-dimensional dynamic equations of Koiter's shell.

On the Stationary Solution of the Mathematical Model for Grain Boundary Grooving

OGASAWARA[♯] AND AKIRA YANAGIYA[†]
[♯]*Science and Engeneering Department, Waseda University, Japan*
[†]*Advanced Institute for Complex Systems, Waseda University, Japan*
E-mail: [♯]609h5039@mn.waseda.ac.jp and [†]yanagiya@waseda.jp

---

In this talk, we will present some stationary solution for nonlinear partial differential equation called Mullins Equation which occurs in the theory of grain boundary grooving:

$$u_t = -C_1^E(u)(1+u_x^2)^{1/2} exp(-C_2^E(u)\frac{u_{xx}}{(1+u_x^2)^{3/2}}) + C_1^C(u)(1+u_x^2)^{1/2}.$$

The main tool, which we can use, is the admissibility property between weighted continuous function spaces for the integral operator as follows:

$$T_\xi x(t) = -\int_t^\infty e^{\zeta_1(t-s)} F(x(s), y(s))ds,$$

$$T_\xi y(t) = \xi e^{\zeta_2 t} + \int_0^t e^{\zeta_2(t-s)} F(x(s), y(s))ds.$$

From this admissibility we can prove the existence theorem for the special simultaneous differential equation. This existence theorem can be applied for the second order differential equation,

$$u'' = f(u, u') = \frac{kT(u)(1+u'^2)^{3/2}}{v\gamma} ln(\frac{P_0(u)}{P_c}).$$

The solution of this equation is one of the stationary solution for Mullins Equation.

References

[1]. A. Kitada. *J. Math. Phys.*, (1986), (1987).

[2]. P. Broadbridge. *J. Math. Phys.*, (1989).

High Order Compact Scheme with Multigrid Local Mesh Refinement Procedure for Convection Diffusion Problems

HAIWEI SUN[♯], JUN ZHANG[♯] AND JENNIFER J. ZHAO[†]
[♯]*Department of Computer Science, University of Kentucky, USA*
[†]*Department of Mathematics and Statistics, University of Michigan-Dearborn, USA*
E-mail: [†]xich@umd.umich.edu

We derive a new fourth order compact finite difference scheme which allows different meshsize in different coordinate directions for the two dimensional convection diffusion equation. A multilevel local mesh refinement strategy is used to deal with the local singularity problem. A corresponding multilevel multigrid method is designed to solve the resulting sparse linear system. Numerical experiments are conducted to show that the local mesh refinement strategy works well with the high order compact discretization scheme to recover high order accuracy for the computed solution. Our solution method is also shown to be effective and robust.