Response to "Comment on 'Surface electromagnetic wave equations in a warm magnetized quantum plasma'" [Phys. Plasmas 23, 074701 (2016)]

Chunhua Li, Zhengwei Wu, Weihong Yang, and Paul K. Chu

1Department of Modern Physics, University of Science and Technology of China, 230026 Hefei, China
2Department of Physics and Materials Science, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong
3Center of Low Temperature Plasma Application, Yunnan Aerospace Industry Company, Kunming, 650229 Yunnan, People’s Republic of China

(Received 15 February 2016; accepted 29 June 2016; published online 22 July 2016)

By reviewing the previous work [C. Li et al., Phys. Plasmas 21, 072114 (2014)] and the Comment of Moradi, some errors are found. Also, an erratum is given in this Response. Published by AIP Publishing. [http://dx.doi.org/10.1063/1.4958656]

In the received Comment,1 the author derived a new dispersion relation, which is similar but not identical with Eq. (32) of Ref. 2. We have carefully checked our previous manuscript again and found some errors. Here, we admit the results in the Comment are correct and express our heartfelt thanks to the author. At the same time, we will give an erratum for the original paper in the following.

In the original paper,2 we use weak magnetic limit ($\omega_{ce} \ll \omega$) in the derivation progress, and the basic model remains unchanged. However, Eqs. (18) should be changed into

$$\frac{\partial^2 B}{\partial x^2} - \psi_p^2 B = 0,$$

(18e)

where

$$\psi_p^2 = k_x^2 + \frac{\alpha^2_{pe} - \omega^2}{c^2},$$

(19e)

and Eqs. (20)–(23) can be removed from the original paper. In this case, the magnetic field component and the electric field component for surface waves (see Eqs. (25)–(30)) also change into a new fashion in the following.

As for the magnetic field component, in the plasma region ($x > 0$)

$$B_p = C_p \exp(-\psi_p x),$$

(25e)

and in the vacuum region ($x < 0$) in the form

$$B_e = C_e \exp(\psi_e x),$$

(26e)

where $C_p$ and $C_e$ are integration constant and $\psi_p^2 = k_x^2 - \omega^2/c^2$ is the permeability of a vacuum. As for the electric field component, in the plasma region ($x > 0$)

$$E_p = eA e^{\exp(-\gamma_p x)} \left( \frac{\nu^2_{Fe}}{2} \frac{k_x^2}{4 \nu^2_{Fe}} \right),$$

(27e)

and in the vacuum region ($x < 0$) in the form

$$E_v = R_v \exp(\psi_e x), \quad x < 0,$$

(28e)

where $R_v$ are constant. Consider the electromagnetic field components and appropriate boundary conditions at $x = 0$. Thus, we use the boundary conditions, namely, (i) the tangential component of $E$ and $B$ are continuous at $x = 0$, (ii) the normal component of the displacement vector is continuous at $x = 0$, and (iii) velocity components (along x-axis) vanish for electrons, i.e., $v_x = 0$ at $x = 0$. Thus, after some algebraic manipulations, we can derive the general dispersion relationship for surface waves in such a quantum system as follows:

$$\omega^2 \left( \psi_p \psi_e + \psi_e^2 \right) \left( \omega^2 - \omega^2_{pe} \right) = \omega^2_{pe} k_x^2 (\psi_e - \gamma_p).$$

(32e)

Eqs. (33) can be removed from the original paper. Considering the electrostatic limit ($\epsilon \rightarrow \infty$) and overcritical dense plasma conditions, one can derive the solutions of Equation (32e)

$$\omega^2 = \frac{\omega^2_{pe}}{2} \left[ 1 + \frac{\omega_{ce}^2}{\sqrt{\omega^2_{pe} - \omega^2 + k_x^2 \nu_{Fe}^2}} \sqrt{\frac{\omega^2_{pe}}{\nu_{Fe}^2} + \frac{h^2 k_x^2}{4m_e^2}} \right].$$

(33)

By ignoring the effects caused by the quantum Bohm potential effects, that is, $\hbar \rightarrow 0$, Equation (33) gives the dispersion relation as

$$\omega = \frac{1}{\sqrt{2}} \sqrt{\left( \omega^2_{pe} + k_x^2 \nu_{Fe}^2 \right) \pm k_y \nu_{Fe} \sqrt{2 \omega^2_{pe} + k_x^2 \nu_{Fe}^2}}.$$

(34)

If we also ignore the Fermi statistical pressure, that is, $\nu_{Fe} \rightarrow 0$, Equation (34) will be simplified as

$$\omega = \frac{\omega_{pe}}{\sqrt{2}}.$$  

(35)

This is just the classical non-dispersive surface plasmon frequency.3

By the way, we point out that if we consider weak magnetic limit ($\omega_{ce} \ll \omega$) and ignore all quantum effects in the dispersion relation obtained by Moradi (see Eqs. (21) in Ref. 1), that is,

$$\omega^2_{pe} [\omega + \omega_{ce}] = 2 \epsilon_x^2 \left| y + k_y \right| \left| \gamma \omega - \omega_{ce} k_y \right|,$$

we can derive the same results like equations in the above.

1Author to whom correspondence should be addressed. Electronic mail: wuzw@ustc.edu.cn
Eqs. (36) should be removed from the original paper, meanwhile, Figs. 4–6 of Ref. 2 will also be removed.

The author would like to thank Dr. Haijun Ren for his helpful discussions and suggestions.

Surface electromagnetic wave equations in a warm magnetized quantum plasma

Chunhua Li,1 Zhengwei Wu,1,2,3,a) Weihong Yang,1 and Paul K. Chu2
1Department of Modern Physics, University of Science and Technology of China, 230026 Hefei, China
2Department of Physics and Materials Science, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong
3Center of Low Temperature Plasma Application, Yunnan Aerospace Industry Company, Kunming, 650229 Yunnan, People’s Republic of China

(Received 6 April 2014; accepted 30 June 2014; published online 11 July 2014)

Based on the single-fluid plasma model, a theoretical investigation of surface electromagnetic waves in a warm quantum magnetized inhomogeneous plasma is presented. The surface electromagnetic waves are assumed to propagate on the plane between a vacuum and a warm quantum magnetized plasma. The quantum magnetohydrodynamic model includes quantum diffraction effect (Bohm potential), and quantum statistical pressure is used to derive the new dispersion relation of surface electromagnetic waves. And the general dispersion relation is analyzed in some special cases of interest. It is shown that surface plasma oscillations can be propagated due to quantum effects, and the propagation velocity is enhanced. Furthermore, the external magnetic field has a significant effect on surface wave’s dispersion equation. Our work should be of a useful tool for investigating the physical characteristic of surface waves and physical properties of the bounded quantum plasmas.

I. INTRODUCTION

Recently, there has been a rapidly growing interest in physical characteristics of various quantum plasmas1–4 since the quantum plasmas are ubiquitous and have been found in numerous modern sciences and technologies such as astrophysical objects,5 nano-scale electromechanical systems,6 ultra-cold plasmas,7 as well as in intense laser-solid density plasma interaction experiments.8 In addition, the physical characteristics of quantum plasmas have been extensively investigated including the quantum electron tunneling effects described by Bohm potential term and the Fermi-Dirac quantum statistical pressure.9 In quantum plasmas, when the de Broglie wavelength of the charged carriers become comparable to the dimensions of the system (such as inter-particle distances), quantum effects start playing a role. In the quantum regime, the plasma obeys certain conditions, as discussed by Manfredi.1 There are three well-known models to describe quantum plasma systems, the Wigner-Poisson (WP) model (in the presence of magnetic fields, the so-called Wigner-Maxwell model), Hartree statistical model, and quantum hydrodynamic (QHD) model. The WP model describes the statistical behavior of quantum plasmas, whereas the Hartree model describes the hydrodynamic behavior.1 As a fluid model, the QHD model has it’s advantage of mathematical efficiency and can be derived from the WP model and/or the Hartree model.2,3 It has been extensively used in the study of quantum plasmas transport, waves and instabilities. The quantum magnetohydrodynamic (QMHD) model has also been obtained using the QHD model with magnetic field based on the Wigner-Maxwell equations.

In the past few years, there has been a great interest in the properties of surface waves in classical and quantum plasmas.10–13 The reason is that all plasmas are in some fashion bounded. When studying the interaction of electromagnetic fields with plasma particles, it is necessary to take into consideration not only the bulk or volume modes, but also the modes localized on the plasma surface, i.e., the so-called surface waves.14 The first serious investigation of surface waves on plasma columns was carried out by Trivelpiece and Gould.11 The authors restricted their attention to electrostatic surface waves on cold cylindrical plasma columns. Kaw and Mcbride15 investigated analytically the effect of density gradients and a finite temperature on the dispersion relation for surface waves on a plasma half-space. They found that when the density variation over a wavelength is very large, a new type of damped surface wave with a frequency higher than the surface plasma frequency is possible. Vedenov16 gave a general dispersion relation for the case of a semi-infinite plasma with a plane boundary, which showed that surface waves exist with a certain range of frequencies. The propagation of electrostatic surface waves on the plane interface between a vacuum and a cold magnetized homogeneous plasma has already been investigated.17 Surface modes can be used for plasma diagnostics.18 Furthermore, surface waves are relevant to laser fusion and the solar corona.19,20

In most of the aforementioned works, surface waves are assumed to propagate whenever there is a boundary between plasma and a dielectric medium or vacuum. The linear theory of surface wave propagation has been studied by assuming an interface between plasma and vacuum, each of which occupies a semi-infinite half-space, for a variety of situations including cold, warm, unmagnetized plasmas.13

With the development of quantum plasma physics, the propagation of surface waves on a quantum plasma half-

---

a)Author to whom correspondence should be addressed. Electronic mail: wuzw@ustc.edu.cn
space has been investigated in many works. The quantum surface waves on a thin plasma layer and their instabilities have been studied by Shokri and Rukhadze. They have shown that in the presence of quantum effects, the surface waves are unstable in a two-component quantum plasma, whereas they damp in a one-component plasma. Theoretical study of the surface waves in semi-bounded quantum plasmas with the effects of spatial dispersion and collisions has been investigated. Recent works showed that the plasma spatial dispersion significantly affects the properties of surface waves, especially at short wavelengths. Furthermore, the surface waves can be unstable in the presence of collisional effects. The relativistic effects and electron exchange-correlation effects on the surface wave are studied numerically and analytically, respectively. The quantum effects and slab geometric effects on the symmetric and anti-symmetric surface modes are discussed by Jung and Hong.

In addition, the propagation of surface waves in a magnetized quantum plasma has been a topic of important research. Many physicists have published relevant papers about it based on various models. The dispersion properties of a transverse electric (TE) surface waves propagating along the interface between a relativistic (nonrelativistic) quantum plasma system and vacuum are studied by using the QHD model, and it is found that the quantum effects play a crucial role to facilitate the propagation of TE surface waves. To the best of our knowledge, the investigation of surface wave propagation on a plasma-dielectric interface may be done in two ways: one suppose that the surface wave arises from a perturbation which propagates. We then follow the time development of an initial perturbation. The finding of the dispersion relation reduces to solving the initial problem for bounded plasma. The other is to calculate the index of refraction of an electromagnetic wave on the plasma-dielectric boundary, in this way, one can obtain the condition for the propagation of a surface wave on the interface. Most of the theoretical works devoted to the study of surface wave propagation in a plasma, in which the former method is often used.

Quite recently, the problem of surface electromagnetic waves (SEMWs) has been attracted much attention. They are of practical interest because their energy decreases in inverse proportion to the distance from a point-like source, while the energy of bulk electromagnetic waves decreases in inverse proportion to the distance squared to the source. It is well known that surface electromagnetic waves can be excited at the interface between two or more media. These two-dimensional electromagnetic waves die down exponentially along the third coordinate. The high-frequency surface electromagnetic waves in a semi-bounded weakly ionized plasma are studied in Ref. They found that the high-frequency surface waves become strongly damped at a certain range of wavelength. In 2005, de Abajo and Saenz had observed the electromagnetic surface modes in structured perfect-conductor surfaces, and the investigations shown that the chosen metamaterials cannot be described in general by local, frequency-dependent permittivities and permeabilities for small periods compared to the wavelength except in certain limiting cases. Also, the propagation of surface electromagnetic waves along a uniform magnetic field was studied in a quantum electron-hole semiconductor plasma. A forward propagating mode is found by including the effect of quantum tunneling. However, many aspects of excitation and propagation of the SEMWs still remain uninvestigated. Thus, in the present work, due to the QMHD model, we investigate the propagation of SEMWs on a warm quantum electron-ion plasma-vacuum interface. We consider the wave’s magnetic field which is parallel to the interface, corresponding to a transverse magnetic (TM)-wave. The quantum effects are described by quantum diffraction effect (described by Bohm potential) and Fermi statistical pressure.

This paper is outlined in the following way: In Sec. II, the assumptions and basic equations are presented. In Sec. III, we devote to obtain the dispersion relation and the corresponding discussion. Finally, a summary and conclusion are given in Sec. IV.

II. ASSUMPTIONS AND EQUATIONS

We suppose that the quantum plasma is composed of electrons and ions. The ions are assumed to be stationary, and its quantum effects can be ignored since their inertia is too large for them to respond to a high-frequency wave. Meanwhile, the quantum plasma half-space (x > 0) is considered to be embedded in an external strong magnetic field \( \mathbf{B}_0 = B_0 \hat{z} \), where \( \hat{z} \) is the unit vector along the Z-axis in a Cartesian coordinate system and \( B_0 \) is the strength of the background magnetic field. A pure surface electromagnetic wave propagates along the plasma-vacuum interface and decays exponentially away from it. Here, we consider surface electromagnetic wave propagation in a semi-infinite plasma, since this geometry gives the basic physical picture of the phenomenon. Also, we assume the vacuum-plasma interface to be sharp, an approximation that is valid for waves whose wavelengths are large compared to the thickness of the transition region (the region where the plasma density is from some finite value to zero and is of the order of quantum debye length).

By using the QMHD model, the basic equations of electromagnetic surface waves are composed of the dynamic equation of an electron as follows:

\[
m_e n_e \frac{\partial \mathbf{u}}{\partial t} = -e n_e (\mathbf{E} + \mathbf{u} \times \mathbf{B}) - \nabla P + \mathbf{F}_Q, \tag{1}
\]

and Maxwell equations

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{3}
\]

\[
\nabla \times \mathbf{B} = \mu_0 \left( \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t} \right), \tag{4}
\]

\[
\nabla \cdot \mathbf{E} = -\frac{e}{\mu_0 n_e}, \tag{5}
\]

\[
\nabla \cdot \mathbf{B} = 0. \tag{6}
\]

Here, we use SI units. \( d/dt = \partial/\partial t + (\mathbf{u} \cdot \nabla) \) is the hydrodynamic derivative, \( m_e \) is electron mass, \( n_e \) is electron number.
density, \(e\) is the magnitude of the electronic charge, \(u\) is the electron fluid velocity, \(E\) and \(B\) are the electric and magnetic field vectors, \(P\) is quantum statistical pressure, \(F_Q = \frac{\hbar^2}{2m_e} \nabla (\nabla^2 \sqrt{n_e} / \sqrt{n_e}) \equiv -\nabla \phi_B\) is quantum force due to electron tunneling through the Bohm potential, \(\phi_B\) represents the Bohm potential, \(\mu_0\) is the magnetic permeability, \(\epsilon_0\) is the vacuum electric conductivity, \(J = -e\n_n u\) is the current density due to free electrons.

We now consider the linear theory of surface wave propagation, that is, we assume that all plasma parameters have the form \(\phi = \phi_0 + \phi_1\), where \(\phi_0\) is the equilibrium part and \(\phi_1\) is the perturbation part owing to the wave field. Without any restrictions on generality, we can assume that the oscillating quantities behave as the form

\[
\phi_1 \propto \phi(x) \exp(-i\omega t + ik_y \cdot y),
\]

where \(\omega\) is the wave frequency and \(k_y\) is the \(y\)-component of the wave vector.

It should be pointed out that the quantum fermionic pressure in the first equation can be written as\(^{35}\)

\[
P = 2T_F n_1.
\]

Here, \(T_F = \frac{1}{4} m_e v_F^2 / k_B = (\hbar^2 / 2m_e)(3\pi^2)^{2/3} n_e^{2/3}\) is the Fermi temperature of degenerate electrons, \(v_F\) is the Fermi velocity, and \(k_B\) is the Boltzmann constant. As for Maxwellian plasma, the Fermi temperature \(T_F\) will be replaced by the Maxwellian temperature \(T_e\), and \(n_1(\ll n_0)\) is a small electron density perturbation.

The first order quantum force \(F_Q\) is\(^{30}\)

\[
F_Q = \frac{\hbar^2}{4m_e} \nabla \nabla^2 n_1.
\]

Equation (9) is the quantum Bohm potential gradient (corresponding to the quantum corrections in the density fluctuations).

Plasma equilibrium is assumed: \(E_0 = 0, u_0 = 0\). The perturbed electric field \(E_1\) and electron fluid velocity \(u_1\) are in the X-Y plane. The wave vector \(k\) is along the Y-axis which is the plasma-vacuum interface. The basic geometry of the model is described in the following graphs (Fig. 1).

Now, we are considering the propagation of surface electromagnetic waves in a warm magnetized quantum plasma, and the basic linearized equations in such quantum plasmas system can be obtained in the following:

\[
m_e n_0 \frac{\partial u_i}{\partial t} = -en_0 (E_1 + u_1 \times B_0) - 2T_F \nabla n_1 + \frac{\hbar^2}{4m_e} \nabla \nabla^2 n_1,
\]

(10)

\[
\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot u_1 = 0,
\]

(11)

\[
\nabla \times E_1 = -\frac{\partial B_1}{\partial t},
\]

(12)

\[
\nabla \times B_1 = \mu_0 \left( J_1 + \epsilon_0 \frac{\partial E_1}{\partial t} \right),
\]

(13)

\[
\nabla \cdot E_1 = -\frac{e}{\epsilon_0} n_1.
\]

(14)

Using the linearized Eqs. (10) and (11), one can obtain the wave equation for the density perturbation as follows:

\[
\omega^2 n_1 = \omega_{pe}^2 n_1 - v_F^2 \frac{\partial^2 n_1}{\partial x^2} + v_F^2 k_y^2 n_1 + \frac{\hbar^2}{4m_e} \left( \frac{\partial^2 n_1}{\partial x^2} - 2k_y^2 \frac{\partial^2 n_1}{\partial y^2} + k_y^4 n_1 \right),
\]

(15)

where \(\omega_{pe} = \sqrt{e^2 n_0 / \epsilon_0 m_e}\) and \(v_F = \sqrt{2k_B T_F / m_e}\) are the electron plasma frequency and the electron Fermi velocity, respectively.

The very slow nonlocal variations in Eq. (15) can be neglected, (i.e., \(k_y^2 \partial^2 / \partial x^4 \ll \partial^2 / \partial x^2 \ll k_y^2\)). Then, Eq. (15) yields the following wave equation for the perturbed electron density:

\[
\frac{\partial^2 n_1}{\partial x^2} - \gamma_p^2 n_1 = 0,
\]

(16)

where

\[
\gamma_p^2 = \frac{\left( \omega_{pe}^2 - \omega^2 \right) + k_y^2 v_F^2 + \frac{\hbar^2 k_y^2}{4m_e}}{v_F^2 + \frac{\hbar^2 k_y^2}{2m_e}}.
\]

(17)

We focus on the propagation and oscillation of transverse magnetic surface modes, assuming the electromagnetic field perturbation has the form \(E_x, E_y, B_z\), from Maxwell’s equations, one obtains the wave equation of the magnetic field component for these surface waves in the following:

\[
\frac{\partial^2 B_z}{\partial x^2} - \psi_p^2 B_z = \chi_p^2 \frac{\partial n}{\partial x},
\]

(18)

where

\[
\psi_p^2 = k_y^2 \left( \frac{\omega^2 - \omega_{pe}^2}{\omega^2} \right),
\]

(19)

\[
\chi_p^2 = \frac{ek_y v_F^2}{\epsilon_0 c^2 \omega} \left[ 1 - \frac{\omega^2 - \omega_{pe}^2}{\omega^2 - \omega_{He}^2} \right].
\]

(20)
\[ Q = 1 + \frac{\hbar^2 k_y^2}{4m_e^2 v_{Fe}^2}, \]  
\[ \omega_H^2 = \omega_{pe}^2 + \omega_{ce}^2, \]  
\[ \omega_{ce} = \frac{eB_0}{m_e}. \]  
Here, \( \omega_H \) and \( \omega_{ce} \) are the upper hybrid frequency and Larmor frequency, respectively.

Equation (16) has the following finite solution inside the quantum magnetized plasma region \( (x > 0) \) as:

\[ n_1 = A_0 \exp(-\gamma_p x), \]  
\[ \exp(-\gamma_p x) + C_1 \omega_c^2 \psi_v \exp(-\psi_v x) \], \( x > 0, \]  
\[ \exp(-\gamma_p x) + C_2 \omega_c^2 \psi_v \exp(-\psi_v x) \], \( x < 0. \]  

Next, from the continuity condition of the electromagnetic field components through the interface plane in both regions as well as the appropriate boundary condition at \( x = 0 \) for the electron plasmas, we can get the amplitudes \( C_1 \) and \( C_2 \) of the electromagnetic field at the two regions

\[ C_1 = \frac{A_0}{\psi_v + \psi_p} \left[ \gamma_p^2 \zeta_p^2 \left( \frac{\gamma_p + \psi_v}{\gamma_p^2 - \psi_v^2} \right) - \frac{e k_y^2 v_{Fe}^2}{\epsilon_0 \omega_c^2 \zeta_p^2} \right], \]  
\[ C_2 = \frac{A_0}{\psi_v + \psi_p} \left[ \gamma_p^2 \zeta_p^2 \left( \frac{\gamma_p + \psi_v}{\gamma_p^2 - \psi_v^2} \right) - \frac{e k_y^2 v_{Fe}^2}{\epsilon_0 \omega_c^2 \zeta_p^2} \right]. \]  

### III. DISPERSION RELATIONS AND DISCUSSIONS

In order to get a dispersion relation for the surface electromagnetic waves, it is necessary to apply the electromagnetic boundary conditions on the wave field at the interface between the two media, namely, the continuity of the tangential components of the electric and magnetic fields. These conditions can be obtained by integrating Maxwell’s equations across the interface, that is,

\[ B_{z,\text{surface}} = B_{z, \text{t}}, \]  
\[ E_{\text{zyt, surface}} = E_{\text{zyt, t}}. \]  

where the subscript \( t \) stands for the direction of tangential.

Also, by using Maxwell equations and the velocity components (parallel to X axis) vanish for both electron plasmas and vacuum, in other words, \( u_e = 0 \) (at \( x = 0 \)), after some complicated algebraic manipulations, we can derive the general dispersion relationship for surface electromagnetic waves in such quantum systems. By taking into account the expressions of \( \gamma_p \) and \( \zeta_p \), we then obtain the following simplified dispersion relation:

\[ (\omega^2 - \omega_{pe}^2) \chi^2 = \omega_{ce}^2 \psi_v + (\omega^2 - \omega_H^2) \psi_p^2, \]  
where

\[ \chi^2 = \frac{(\omega_{pe}^2 - \omega^2) + k_y^2 v_{Fe}^2 Q}{2(Q - 1)}. \]  

Equation (32) is the dispersion relation describing the propagation of TM surface waves in the warm quantum magnetized plasma. And, it can be used to investigate at some representative astrophysical systems which will be discussed later. It has been demonstrated that the surface electromagnetic waves arising in a plasma layer confined by a strong external magnetic field can be strongly affected by the quantum effects. Nevertheless, Eq. (32) indicates that taking into account the external magnetic field, the dispersion behavior is changed.

Since it is difficult to find a general solution of Eq. (30) analytically, we then discuss and investigate Eq. (32) in some special cases of interest. First, in the absence of the Bohm potential term, namely, \( \hbar \to 0 \), it is reduced to the following equation:

\[ \omega^2 = \omega_{pe}^2 + k_y^2 v_{Fe}^2. \]  

Equation (34) indicates that surface plasma oscillations can be propagated due to Fermi statistical pressure. It is noticed that if all the quantum effects are ignored, namely, \( v_{Fe} \to 0 \), Eq. (34) will become to the well known surface Langmuir oscillation

\[ \omega = \omega_{pe}. \]  

Here, if we restrict our attention to the quantum effects, we can also assume the factor \( \omega_{ce} = 0 \) for the sake of simplicity. One can then get the following relation from Eq. (32):
Equation (36) can be used to investigate the propagation characteristic of surface waves in non-magnetic quantum plasmas.

Next, in order to show the influence of the quantum effects due to Bohm potential and Fermi statistical pressure, we evaluate Eqs. (34) and (36) by substituting some typical parameters in the dense astrophysical objects (like the outer shells of magnetized white dwarf stars), where the plasma density is \( n_0 = 10^{34} \text{ m}^{-3} \).

Figure 2 corresponds to the propagation of surface Langmuir oscillation due to the quantum statistical pressure in a certain range of wave numbers. The regime of its propagation is due to the dynamical motion of the electrons, which introduces a restoring force that causes the propagation of oscillation. It is also the influence of the quantum statistical pressure on the dynamics of our quantum system.

Figure 3 describes the comparison of quantum statistical pressure effect (blue line) to the Bohm potential which is represented by the red line. The effect of quantum Bohm potential correction on the dispersion relationship of surface electromagnetic waves is apparent. From this figure, we also found that including both the Bohm potential effect and quantum statistical pressure effect, the group velocity, namely, the slope of the dispersion curve, increases with increasing of wave number in a certain range of wave numbers.

Meanwhile, by taking typical parameter values in the dense astrophysical objects (in the vicinity of pulsars and magnetars), where the strength of magnetic field is about \( 10^7 \text{ T} \), we also explore the change of the dispersion relation due to the variation of the magnetic field strength and give corresponding figure with its discussions in the following.

We see from Figs. 4–6 that the influence of the external magnetic field on the surface electromagnetic waves

\[
(\omega^2 - \omega_{pe}^2)[2(Q - 1) - c^2] = k^2 c_s^2[2(Q - 1) - v_F^2 Q].
\]

FIG. 2. Plot of frequency versus the wave number for the propagation of surface wave on a quantum plasma-vacuum interface (only the contribution of quantum statistical pressure effect is considered). The range of wave number \( k \) is between \( 2 \times 10^9 \text{ m}^{-1} \) and \( 2 \times 10^{11} \text{ m}^{-1} \).

FIG. 3. Plot of frequency versus the wave number for the propagation of surface wave on a quantum plasma-vacuum interface (contribution of quantum statistical pressure effect and Bohm potential are included). The blue line is for the quantum statistical pressure effect only, and the red line is considering both the Bohm potential effect and quantum statistical pressure effect. The range of wave number \( k \) is between \( 4 \times 10^{11} \text{ m}^{-1} \) and \( 4 \times 10^{12} \text{ m}^{-1} \).

FIG. 4. Plot of frequency versus the wave number for the propagation of surface wave on a dense quantum plasma-vacuum interface, where the strength of magnetic field is \( 10^7 \text{ T} \).

FIG. 5. Plot of frequency versus the wave number for the propagation of surface wave on a dense quantum plasma-vacuum interface, where the strength of magnetic field is \( 3 \times 10^7 \text{ T} \).

FIG. 6. Plot of frequency versus the wave number for the propagation of surface wave on a dense quantum plasma-vacuum interface, where the strength of magnetic field is \( 5 \times 10^7 \text{ T} \).
dispersion relation is apparent in a certain range of wave number. With the increase of magnetic field intensity, the surface electromagnetic wave’s frequency is changed for some fixed wave numbers.

IV. CONCLUSION

The propagation of electromagnetic surface waves in warm magnetized quantum plasmas is investigated. A new general dispersion relation of surface electromagnetic waves is derived. Our results present that quantum effects can play an important role of Larmor oscillations. If the quantum effects are neglected, there are only Langmuir oscillations without propagation as in the case of classical plasma. In other words, quantum effects play the same role of thermal effects to facilitate the Langmuir oscillations. Meanwhile, the external magnetic field modifies the dispersion relation a lot. It is clear from the corresponding figure that the group velocity increases in a certain range of wave numbers. One of an important result of this study is the influence of the external magnetic field on the surface electromagnetic waves dispersion equations. Our work should be of a useful tool for investigating the physical characteristic of quantum surface waves and physical properties of the bounded quantum magnetic or nonmagnetic plasmas. The results may be significant for understanding the dispersion properties of new quantum surface waves in plasmas, which can be observed experimentally in the near future.

ACKNOWLEDGMENTS

One of the authors (Chunhua Li) would like to thank Dr. Chao Dong, Zhenwei Xia, and Wenlong Huang for their valuable suggestions and comments. This project was supported by the Chinese National Natural Science Foundation under Grant Nos. 11375190 and 11075163.

30B. F. Mohamed, Phys. Scr. 82, 065502 (2010).