Magnetothermal instability in weakly magnetized plasmas with anisotropic resistivity and viscosity

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The linear buoyancy instability in magnetized plasmas is investigated in the presence of anisotropic resistivity and viscosity. The magnetic field is assumed to be horizontal and the background heat flux is not taken into account. That is, the magnetic field lines are initially isothermal. The heat is assumed to be primarily transported along the magnetic force lines. The general dispersion relationship of the thermal convective instability in the presence of anisotropic resistive and viscous dissipative effects is derived and discussed in detail in weak magnetic field limit. Our results show that the perturbation is damped when the temperature decreases in the direction of gravity due to the resistive or viscous effect while this situation results in pure oscillation modes in the ideal MHD case. The resistive and viscous effects are shown to reduce the growth rate of the magnetothermal instability when the temperature increases in the direction of gravity. © *2010 American Institute of Physics*. [doi:10.1063/1.3398478]

I. INTRODUCTION

Thermally stratified fluids are shown to be buoyantly unstable when the thermal temperature increases in the direction of gravity in a low-collisional plasma immersed in a weak magnetic field.[1,2] This is the so-called magnetothermal instability (MTI). The heat flux in such a plasma is primarily along the magnetic force lines when the mean free path between ion collisions is much greater than the ion Larmor radius.[3] In this case, the anisotropic transport terms must be taken into account by the magnetohydrodynamic (MHD) equations to describe the dynamic behavior of plasmas.[1,4–10] A simple case in which there exists a horizontal magnetic field in a vertically stratified plasma in the absence of the background heat flux was investigated by Balbus.[1]

In a following paper, Balbus investigated the kind of thermal convective instability of a dilute plasma in the presence of rotating flows while still not taking into account the heat flux in the background plasma.[2] Parrish and Stone used numerical methods to explore the nonlinear evolution and saturation of the MTI in two dimensions. They showed that the linear growth rates measured in the simulation agreed well with the weak-field dispersion relation.[4] Later, the saturation and heat transport properties of the MTI in three dimensions is presented in Ref. 5. After that, Quataert[11] calculated the linear instability of a stratified low-collisional and weakly magnetized plasma. The magnetic field was assumed to have both vertical and horizontal components. The analytical expression of the MTI in the presence of a heat flux in the background was derived and discussed. His results show that the presence of a heat flux drives a buoyancy instability analogous to the MTI when the temperature decreases in the direction of gravity while this situation is magnetothermally stable according to Balbus' analysis (see Ref. 1). Recently, a three-dimensional MHD simulation of isolated clusters including radiative cooling and anisotropic thermal conduction along the magnetic field lines was investigated by Parrish *et al.*[12]

In the work reported in this paper, we study the stability of local thermal convective mode in weakly magnetized plasmas in the framework of resistive and viscous MHD model. The transverse magnetic field is adopted in our calculation. As aforementioned, we focus on the plasma system in which the mean free path between ions collisions is much greater than the ion gyroradius. In this case, the resistivity and viscosity should be considered as anisotropic too. The general dispersion relationship is derived and shows that anisotropic resistivity and viscosity have significant effects on the MTI. The present paper is organized as follows. The basic equations of MHD model in the presence of anisotropic heat flux and dissipation terms are presented in Sec. II. Based on these equations, the analytical expression of the local linear dispersion relation of MTI is derived. Effects due to the anisotropic resistivity and viscosity on the dispersion relation of the thermal convective instability are discussed in different cases in Sec. III. Finally, Sec. VI is devoted to the discussion and conclusion.

II. BASIC EQUATIONS AND DISPERSION RELATION

The basic set of resistive and viscous MHD equations with the addition of the heat flux, **Q**, and a gravitational field is

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0, \tag{1}$$

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$$\rho\frac{\partial \mathbf{v}}{\partial t} + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + \frac{1}{\mu_0}(\nabla\times\mathbf{B})\times\mathbf{B} + \rho\mathbf{g} + \nabla\cdot\overleftrightarrow{\mathbf{\Pi}}, \tag{2}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E}, \tag{3}$$

$$\rho T\frac{dS}{dt} = -\nabla\cdot\mathbf{Q} + \mathbf{E}\cdot\mathbf{J} + \rho\mathbf{g}\cdot\mathbf{v} + \Pi_{ij}\partial_j v_i. \tag{4}$$

Here, ρ is the mass density, $\mathbf{v}$ is the fluid velocity, p is the pressure, $\mathbf{B}$ is the magnetic field, $\mathbf{g}$ is the gravitation acceleration, $\mathbf{J}=\nabla\times\mathbf{B}/\mu_0$ is the current density, $\overleftrightarrow{\mathbf{\Pi}}$ is the viscosity tensor, $\mathbf{E}$ is the electric field, T is the temperature, and $S=3p\ln(p\rho^{-\gamma})/2\rho T$ is the entropy per unit mass. The adiabatic index γ is 5/3. The plasma is assumed to be thermally stratified in the presence of a uniform gravitational field along the vertical direction, $\mathbf{g}=-g\hat{\mathbf{z}}$. ∂_j is denoted for ∂/∂_{r_j} and summation over repeated suffixes is assumed. Only the transverse magnetic field is initially considered, which means $\mathbf{B}_0=B_x\hat{\mathbf{x}}+B_y\hat{\mathbf{y}}$ without loss of generality. Besides, the initial magnetic field is assumed to be very weak and hence the equilibrium condition is $dp_0/dz=-\rho_0 g$,[11] where p_0 and ρ_0 are the equilibrium isotropic kinetic pressure and mass density, respectively. In this limit, the magnetic forces are dynamically not important. The role of the magnetic field is to enforce an anisotropic transport of heat and to make the resistivity and viscosity anisotropic. Moreover, the general heat-loss function is neglected in our model. Since $\omega_c\tau \gg 1$, where ω_c is the cyclotron frequency and τ is the mean collisional time, the anisotropic heat conducting, resistivity, and viscosity must be used to describe the plasma system. Under this circumstance, the anisotropic heat flux $\mathbf{Q}$ is therefore

$$\mathbf{Q} = -\chi_C\hat{\mathbf{b}}(\hat{\mathbf{b}}\cdot\nabla)T, \tag{5}$$

where χ_C is the Spitzer Coulomb conductivity[13] and $\hat{\mathbf{b}}=\mathbf{B}/B$ is a unit vector in the direction of the magnetic field. The Ohm's law reads[14]

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = \eta_\parallel\mathbf{J}_\parallel + \eta_\perp\mathbf{J}_\perp, \tag{6}$$

where $\eta_\parallel/\eta_\perp=(1+\omega_c^2\tau^2)\gg 1$.[15] Here $\mathbf{J}_\parallel$ and $\mathbf{J}_\perp$ are the current density components that are along and perpendicular to the magnetic field direction, respectively. Since there is $\eta_\parallel\mathbf{J}_\parallel+\eta_\perp\mathbf{J}_\perp=(\eta_\parallel-\eta_\perp)\mathbf{J}_\parallel+\eta_\perp\mathbf{J}$, in which the last term becomes an isotropic term, the main purpose of the present paper is to investigate the effect of anisotropic resistivity on the MTI. Hence we can drop the isotropic resistivity term and rewrite the magnetic diffusing Eq. (3) as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) - \eta\nabla\times[\hat{\mathbf{b}}(\hat{\mathbf{b}}\cdot\nabla\times\mathbf{B})]. \tag{7}$$

For viscosity, it is appropriate to adopt Braginskii's expression for the viscosity tensor $\overleftrightarrow{\mathbf{\Pi}}$,[3,10,16,17]

$$\overleftrightarrow{\mathbf{\Pi}} = \nu_0(3\hat{\mathbf{b}}\hat{\mathbf{b}} - \overleftrightarrow{\mathbf{I}})\left[\hat{\mathbf{b}}\cdot(\hat{\mathbf{b}}\cdot\nabla)\mathbf{v} - \tfrac{1}{3}\nabla\cdot\mathbf{v}\right], \tag{8}$$

where ν_0 is the first Braginskii's coefficient of viscosity and $\overleftrightarrow{\mathbf{I}}$ is the unit tensor.

In this work, we consider a simple case in which $\mathbf{B}_0\cdot\nabla T=0$. This implies that the heat flux in the initial state is zero, that is, no background heat flux, and in the equilibrium condition, the field lines are isothermal. For brevity, we write $\chi_C=\chi$ and $\eta=\eta_\parallel/\mu_0$. The perturbed profile $\delta\varphi$ is assumed to have the Wentzel–Kramers–Brillouin (WKB) space-time dependence $\exp(-i\omega t+i\mathbf{k}\cdot\mathbf{r})$, where ω is the wave frequency and $\mathbf{k}$ $(=k_x\hat{\mathbf{x}}+k_z\hat{\mathbf{z}})$ is the wave vector. The perturbations are supposed to be independent of y. The WKB assumption requires $kL\gg 1$, where L is the local scale length of the system and $k=|\mathbf{k}|$ is the total wave number. Furthermore, it is sufficient for us to work in the Boussinesq limit (i.e., the fluctuation in pressure is much less than that in density, viz., $\delta p/p_0\ll\delta\rho/\rho_0$, and in the perturbed mass conservation and momentum equations, the density variations can be neglected except when they are coupled to the gravitational term).[18,19] In this case, one gets $\partial_i v_i=0$ and hence, the viscosity term in Eq. (4) is simplified to

$$\begin{aligned}\Pi_{ij}\partial_j v_i &= \nu_0(3b_ib_j-\delta_{ij})b_mb_k\partial_k v_m\partial_j v_i\\ &= \nu_0(3b_ib_j\partial_j v_i b_m b_k\partial_k v_m) - \nu_0 b_m b_k\partial_k v_m\partial_i v_i\\ &= 3\nu_0[\mathbf{b}\cdot(\mathbf{b}\cdot\nabla)\mathbf{v}]^2.\end{aligned} \tag{9}$$

Then linearizing Eqs. (1)–(4) by means of Eqs. (5), (7), and (8), we then obtain

$$k_x\delta v_x + k_z\delta v_z = 0, \tag{10}$$

$$\begin{aligned}-i\omega\rho_0\delta\mathbf{v} = &-i\mathbf{k}\delta p + \delta\rho\mathbf{g} + \frac{i}{\mu_0}(\mathbf{k}\cdot\mathbf{B}_0)\delta\mathbf{B}\\ &-\frac{i}{\mu_0}\mathbf{k}(\mathbf{B}_0\cdot\delta\mathbf{B})\\ &+\nu_0(\hat{\mathbf{b}}\cdot\delta\mathbf{v})(\mathbf{k}\cdot\hat{\mathbf{b}})[\mathbf{k}-3\hat{\mathbf{b}}(\mathbf{k}\cdot\hat{\mathbf{b}})],\end{aligned} \tag{11}$$

$$-i\omega\delta\mathbf{B} = \frac{\mathbf{B}_0\cdot i\mathbf{k}}{\eta\Theta^2 - i\omega}[-i\omega\delta\mathbf{v} - \eta\mathbf{\Theta}\times(\mathbf{\Theta}\times\delta\mathbf{v})], \tag{12}$$

$$\begin{aligned}\left[i\omega - \frac{2\chi T}{5p_0}(\mathbf{k}\cdot\hat{\mathbf{b}})^2\right]\frac{\delta\rho}{\rho} = &\frac{2\chi T}{5p_0B_0}[(i\mathbf{k}\cdot\hat{\mathbf{b}})(\delta\mathbf{B}\cdot\nabla\ln T)]\\ &-\frac{3}{5}\delta\mathbf{v}\cdot\nabla\ln p_0\rho_0^{-\gamma} - \frac{2\rho_0}{5p_0}g\delta v_z.\end{aligned} \tag{13}$$

Here $\mathbf{\Theta}=\mathbf{k}\times\hat{\mathbf{b}}$ and $\Theta^2=k^2b_y^2+k_z^2b_x^2$ with $b_x=B_x/B_0$ and $b_y=B_x/B_0$. To derive Eq. (13), we have adopted $\delta\hat{\mathbf{b}}=\delta\mathbf{B}/B_0-\hat{\mathbf{b}}(\delta B/B_0)$ and $\delta T\simeq -T\delta\rho/\rho_0$.

The perturbed magnetic fields are derived from the perturbed magnetic diffusing Eq. (12) as

$$\delta B_x = \frac{\mathbf{k}\cdot\mathbf{B}_0}{i\omega - \eta\Theta^2}[(\eta b_x^2 k_z^2 - i\omega)\delta v_x + \eta b_x b_y k_z^2 \delta v_y],$$

$$\delta B_y = \frac{\mathbf{k}\cdot\mathbf{B}_0}{i\omega - \eta\Theta^2}[(\eta b_y^2 k^2 - i\omega)\delta v_y + \eta b_x b_y k^2 \delta v_x], \tag{14}$$

$$\delta B_z = \frac{\mathbf{k}\cdot\mathbf{B}_0}{i\omega - \eta\Theta^2}[(\eta b_x^2 k_z^2 - i\omega)\delta v_z - \eta b_x b_y k_x k_z \delta v_y].$$

Substituting the perturbed magnetic fields and Eq. (10) into Eq. (11), we obtain the perturbed mass density $\delta\rho$ and δB_z in terms of δv_z, i.e., the parallel (with respect to the gravitational field) component of the perturbed velocity,

$$\delta\rho = \left[\frac{i(i\omega^{*2} - \omega\eta\Theta^2)(-i\omega^{*2}k^2 + 3\omega_\nu\omega\Theta^2)}{-i\omega^{*2}(i\omega - \eta\Theta^2) + i\eta k_z^2 b_x^2(\omega^2 - \omega^{*2}) + 3\omega_\nu\omega b_y^2(i\omega - \eta\Theta^2)}\right]\frac{\rho_0}{g k_x^2}\delta v_z \tag{15}$$

and

$$\delta B_z = (\mathbf{k}\cdot\mathbf{B}_0)\left[\frac{-i\omega\eta k_z^2 b_x^2 - \omega^{*2} - i3\omega\omega_\nu b_y^2}{-i\omega^{*2}(i\omega - \eta\Theta^2) + i(\omega^2 - \omega^{*2})\eta k_z^2 b_x^2 + 3\omega\omega_\nu b_y^2(i\omega - \eta\Theta^2)}\right]\delta v_z, \tag{16}$$

where we have denoted $\omega^{*2} = \omega^2 - (\mathbf{k}\cdot\mathbf{v}_A)^2$ with $\mathbf{v}_A = \mathbf{B}_0/(\rho\mu_0)^{1/2}$ the local Alfvén speed and $\mathbf{k}\cdot\mathbf{v}_A$ the Alfvén frequency. $\omega_\nu = \sigma_0(\mathbf{k}\cdot\hat{\mathbf{b}})^2$ is the viscosity frequency with $\sigma_0 = \nu_0/\rho_0$ being the kinetic viscosity coefficient.

Substituting Eqs. (15) and (16) into Eq. (13), we arrive at the dispersion relationship of thermal convective instability after some algebraic manipulation,

$$(\omega + i\omega_T)(\omega^{*2} + i3\omega\omega_\nu\Theta^2/k^2) - \varpi^2\frac{k_x^2}{k^2}\omega\left[1 + \frac{(\omega^2 - \omega^{*2})\eta k^2 b_y^2 + i3\omega\omega_\nu b_y^2(i\omega - \eta\Theta^2)}{i\omega\omega^{*2} - \eta\Theta^2\omega^2}\right] - i\omega_T g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T\frac{\omega^{*2} + i\omega\eta k_z^2 b_x^2 + i3\omega\omega_\nu b_y^2}{\omega^{*2} + i\omega\eta\Theta^2} = 0, \tag{17}$$

where

$$\varpi^2 = -g\frac{d}{dz}\ln\rho_0 + g\frac{3}{5}\frac{d}{dz}\ln p_0 + \frac{2\rho_0 g^2}{5p_0}, \tag{18}$$

$$\omega_T = \frac{2}{5}\frac{\chi T}{p_0}(\mathbf{k}\cdot\hat{\mathbf{b}})^2. \tag{19}$$

The first and the second terms on the right-hand side of Eq. (18) represent the inhomogeneity of the mass density and pressure, respectively. The last term represents the dissipation due to the gravitational effect. ω_T represents the heat conducting effect. The anisotropic resistive effects are represented by the η-dependent terms and ω_ν-dependent terms represent the anisotropic viscosity effects. Equation (17) is the general dispersion relationship of the MTI with constant buoyancy force effect in the presence of anisotropic resistivity and viscosity in a thermally stratified plasma. Letting $\eta=0$ and $\omega_\nu=0$, our result (17) is reduced to the earlier one reported in Ref. 11. For weak magnetic field limit, i.e., for $\omega^{*2} \simeq \omega^2$, the dispersion relation is simplified to

$$(\omega + i\omega_T)\omega - \varpi^2\frac{k_x^2}{k^2}\frac{\omega + i3\omega_\nu b_y^2}{\omega + i3\omega_\nu\Theta^2/k^2} - i\omega_T g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T\frac{\omega + i\eta k_z^2 b_x^2 + i3\omega_\nu b_y^2}{(\omega + i3\omega_\nu\Theta^2/k^2)(\omega + i\eta\Theta^2)} = 0. \tag{20}$$

III. ANALYSIS

The dispersion relationship given in Eq. (20) is a quartic equation about the frequency ω. It is easy to find that when we let ω_T equal zero, which means that anisotropic thermal conduction is ignored, we arrive at

$$\omega^2 = \varpi^2\frac{k_x^2}{k^2}\frac{\omega + i3\omega_\nu b_y^2}{\omega + i3\omega_\nu\Theta^2/k^2} \tag{21}$$

immediately. The formula above provides the dispersion relationship for classical hydrodynamic convection and internal gravity instability in the presence of anisotropic viscosity.[20,21] The reduced dispersion relation (21) indicates that anisotropic resistive has no effect on the kind of instability in the weak magnetic field limit. Now we concentrate on the local thermal convective instability in the presence of anisotropic resistivity and viscosity by restricting ourselves to the assumption that the perturbation frequencies of interest have the following ordering relationship: $\omega_T \gg \omega_d$, where $\omega_d \sim \sqrt{g/L}$ is the local dynamical frequency. We stress that this is just a scaling relationship providing a characteristic frequency to such perturbations (e.g., see Ref. 11), viz., $\omega \sim \omega_d$, but not the exact numerical expression. Under the weak inhomogeneity approximation, this time scale ordering relationship can always be satisfied. With this time scale hierarchy, the dispersion relationship is reduced to

$$\omega - g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T\frac{\omega + i\eta k_z^2 b_x^2 + i3\omega_\nu b_y^2}{(\omega + i3\omega_\nu \Theta^2/k^2)(\omega + i\eta\Theta^2)} = 0. \quad (22)$$

Now letting $\omega_\nu=0$ in Eq. (22) yields

$$\omega^2 - g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T\frac{\omega + i\eta k_z^2 b_x^2}{\omega + i\eta\Theta^2} = 0. \quad (23)$$

Here we discuss some limiting cases on the basis of the dispersion above. For weak resistivity approximation, i.e., $\eta\Theta^2 \ll \omega_d$, we have

$$\omega = \sqrt{g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T} - i\frac{1}{2}\eta k^2 b_y^2. \quad (24)$$

The above formula describes the MTI discovered by Balbus[1] when we ignore the resistivity. In that case,

$$\omega^2 = g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T. \quad (25)$$

Equation (25) describes a pure oscillation wave mode when $d \ln T/dz > 0$, that is, when the temperature decreases in the direction of gravity as previously analyzed.[1] Equation (24) shows, however, that this is not the case in the presence of resistive effects. For $d \ln T/dz > 0$, the perturbation is damped with a damping rate of $\eta k^2 b_y^2/2$. There is no pure oscillation mode now.

When $d \ln T/dz < 0$, the temperature increases in the direction of gravity. This case has been shown to be magnetothermally unstable.[1,11] Equation (24) implies that the perturbation holds the unstable property and its growth rate Γ $(=i\omega)$ is now

$$\Gamma_{\eta 1} = \sqrt{-g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T} - \frac{\eta k^2 b_y^2}{2}. \quad (26)$$

It is easy to find that $\Gamma < \Gamma_{\text{ideal}}$, where Γ_{ideal} is defined as

$$\Gamma_{\text{ideal}} = \sqrt{-g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T}, \quad (27)$$

which is the growth rate of MTI in ideal MHD case. From the physical point of view, the resistivity enhances the dissipation effect which diminishes the free energy which is supposed to be obtained by the waves. As a result, the growth rate of the perturbations decreases.

When the resistivity becomes dominant, namely, $\eta k_z^2 b_x^2$ is much greater than ω_d, Eq. (23) gives

$$\Gamma_{\eta 2} = \frac{b_x k_z}{\Theta}\sqrt{-g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T} + \frac{k^2 b_y^2}{\Theta^2}\frac{1}{2\eta\Theta^2}\left(-g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T\right). \quad (28)$$

Note that $b_x k_z < \Theta$, this growth rate is still less than Γ_{ideal}. Neglecting the second term on the right-hand side of the formula above, we find that

$$\Gamma_{\eta 2} \simeq \frac{b_x k_z}{\Theta}\Gamma_{\text{ideal}}. \quad (29)$$

The last special case of interest is in which there exists $k_z^2 b_x^2 \ll \Theta^2$, which can be easily satisfied when the perpendicular wave number k_x is much greater than the parallel wave number k_z and/or $b_x \ll b_y$, Eq. (22) yields

$$\Gamma_{\eta 3} = \frac{1}{2}\sqrt{(\eta\Theta^2)^2 - 4g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T} - \frac{1}{2}\eta\Theta^2. \quad (30)$$

From the equation above, we see that the growth rate becomes zero when $\eta\Theta^2 \gg \omega_d$. This is coincident with Eq. (29), which yields $\Gamma_2 \simeq 0$ provided $b_x k_z \ll \Theta$.

Now we concentrate on the viscosity by letting $\eta=0$ in Eq. (22) and then we obtain

$$\omega^2 - g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T\frac{\omega + i3\omega_\nu b_y^2}{\omega + i3\omega_\nu\Theta^2/k^2} = 0. \quad (31)$$

It is clear that the equation above is identical with Eq. (23) when the resistivity terms are replaced by the viscosity terms. Due to the similar expressions of Eqs. (23) and (31), we can directly arrive at some conclusion derived above.

For $\omega_\nu \ll \omega_d$, the dispersion relation (31) goes to

$$\omega = \sqrt{g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T} - i\frac{1}{2}\omega_\nu\frac{k_z^2 b_x^2}{k^2}, \quad (32)$$

which gives the damping rate $\omega_\nu k_z^2 b_x^2/(2k^2)$ for $d \ln T/dz > 0$ and the growth rate as

$$\Gamma_{\nu 1} = \sqrt{-g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T} - \frac{1}{2}\omega_\nu\frac{k_z^2 b_x^2}{k^2}, \quad (33)$$

for $d \ln T/dz < 0$. For $\omega_\nu \gg \omega_d$, we have

$$\Gamma_{\nu 2} = \frac{k b_y}{\Theta}\sqrt{-g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T} + \frac{k_z^2 b_x^2}{\Theta^2}\frac{k^2}{6\omega_\nu\Theta^2}\left(-g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T\right). \quad (34)$$

For $k^2 b_y^2 \ll \Theta^2$, meaning that $b_y \simeq 0$, we find that

$$\Gamma_{\nu 3} = \frac{1}{2}\sqrt{(3\omega_\nu\Theta^2/k^2)^2 - 4g\frac{k_x^2}{k^2}\frac{d}{dz}\ln T} - \frac{3}{2}\omega_\nu\Theta^2/k^2. \quad (35)$$

Our results may have implications for radially stratified atmospheres in which anisotropic transport is present, including atmospheres of strongly magnetized neutron stars,[4] x-ray-emitting gas in clusters of galaxies,[22] and protoplanetary disks.[23] We will apply our results to a standard protoplanetary disks model consisting of a $1M_\odot$ central protostar surrounded by a disk with a mass of $0.091M_\odot$ between 4 and 20 a.u.[24] Taking $d \ln T/dz = -15$ a.u.$^{-1}$, $g = 2.59 \times 10^{-4}$ m s^{-2} for $r = 5$ a.u., $\eta = \sigma_0 = 10^{-4}$ m^2 s^{-1}, $b_x = b_z = 1/\sqrt{2}$, and $k_x = k_z$, then we obtain $\Gamma_{\text{ideal}} = 1.14 \times 10^{-7}$ s^{-1} and $\Gamma_{\eta 1} = 1.09 \times 10^{-7}$ for $k_z = 10^{-2}$ m^{-1}. Note that the condition $\eta\Theta^2 \ll \omega_d$ is hold. For $k \gg 10^{-2}$, Eq. (29) gives $\Gamma_{\eta 2} \simeq \Gamma_{\text{ideal}}/\sqrt{3} = 0.66 \times 10^{-7}$ s^{-1}. These numerical results illustrate that the resistivity effect on the MTI in a stratified plasma is significant. Due to the similarity of expressions, detailed discussion about viscosity effect on the MTI given in Eqs. (33)–(35) is omitted.

The cross-field resistivity is disregarded when we derive Eq. (7) by assuming it is much less than the parallel resistivity. It should be pointed out that the assumption is not suitable for the highly ionized plasmas, in which the well-known

Spitzer resistivity in the cross-field direction is about twice of the parallel resistivity,[25] or more precisely, $\eta_\perp = 1.98\eta_\parallel$ (Ref. 26) when $\omega_c\tau \gg 1$. While in the weakly ionized plasmas, this ordering relation becomes $\eta_\parallel = \omega_c\tau\eta_\perp$.[27] In this case, we can neglect the cross-field resistivity when we apply our result to the protoplanetary disks for a weakly ionized environment.[28]

IV. DISCUSSION AND CONCLUSION

By using the resistive and viscous MHD model, the buoyancy instability in the presence of a weak magnetic field is investigated when the heat flux is assumed to run along the magnetic field lines. The resistivity and viscosity are also considered to be anisotropic. We are restricted to the local dispersion relation by adopting the WKB approximation and the Boussinesq limit. The latter approximation means that the relative changes in the pressure are assumed to be much smaller than those in the temperature or density in our calculation and the continuity equation is replaced by $\nabla \cdot \mathbf{v} = 0$. Equations (17), (20), (22), (26), (28)–(31), and (33)–(35) are the main results of this paper. Equation (17) gives the general dispersion relation of the thermal convective instability in the presence of anisotropic resistive and viscous dissipations and is reduced to Eq. (20) when the magnetic field is assumed to be weak. The reduced dispersion relation of MTI is presented in Eq. (22). The growth rate of MTI for weak resistivity is displayed in Eq. (26) and in Eq. (33) for weak viscosity. When the resistivity (viscosity) becomes dominant, the growth rate is shown in Eqs. (28) and (29) [Eq. (34)]. Finally, Eq. (30) provides the growth rate of resistive MTI when the perpendicular wave number is much greater than the parallel wave number and/or B_x is much less than B_y and Eq. (35) shows the growth rate of viscous MTI for $b_y \simeq 0$. Equations (26), (28), and (30) show that the perturbations are damped when the thermal temperature decreases in the direction of gravity, i.e., when $d \ln T/dz > 0$, while this situation is MTI stable in the absence of resistive effect. It is concluded similarly from Eqs. (33)–(35) that viscosity has damping effect on the perturbations. The growth rate of the MTI is diminished when the temperature increases in the direction of gravity, i.e., when $d \ln T/dz < 0$. In dominant resistivity region, the growth rate is $k_z b_x/\Theta$ times the magnitude of the growth rate in ideal MHD case whereas in dominant viscosity case, the growth rate is kb_y/Θ times the magnitude of the ideal growth rate.

Resistive effects are believed to be very interesting in the presence of internal boundary layers (current sheets) and/or associated with the magnetic reconnection procession.[29] It is known that the resistive regime occurs when collisions between the charge-carrying species and neutrals damp magnetic fields in the plasma. Resistive effects generally become dominant in high density but weakly ionized plasmas.[30] On the other hand, in many astrophysical situations, especially in dilute plasmas, the assumption of ideal MHD, i.e., infinite conductivity so that the magnetic field is frozen-in to the plasma of zero resistivity, does not hold. The plasma in such situations should be characterized by finite resistivity.[31] It seems that there exists a conflict here, but we point out that for sufficient magnetic fields, anisotropic transport is enhanced and meanwhile, plasma density can become greater whereas the ordering relation $\omega_c\tau \gg 1$ can still hold. In this case, anisotropic resistivity must be taken into account by the MHD equations. Furthermore, some astrophysical environments, such as the solar corona, are well-known examples of a plasma which should be described by anisotropic viscous MHD model.[16,17,32] Hence although dissipative processes are small concerning the galaxy clusters and accretion disks, they cannot always be negligible.[33]

It is also believed that in hot accretion flows onto compact objects, there exists $dT/dz < 0$ due to the release of gravitational potential energy and the inflow of matter.[11] Besides, the similar instances can be found in tokamak plasmas (see, for example, Ref. 34) and in the ablative inertial confinement fusion capsules[35,36] in which the temperature increases along the direction of the effective gravity. Under these circumstances, the MTI is arisen and anisotropic dissipative MHD model is suitable for describing the plasmas.

In a word, in the astrophysical disks, when the thermal temperature decreases outward in the presence of inward gravitational acceleration, the heat will be transported along the opposite direction of the gravity and then the disk becomes unstable. On the other hand, it is known that astrophysical disks is a differential rotational system with angular frequency $\Omega = \Omega(r)$, where r is the radii, in which there exists a shear axisymmetric instability. The kind of MHD instability is the so-called magnetorotational instability (MRI),[37,38] which comes into being when the angular velocity decreases outward but the angular momentum increases, therefore resulting angular momentum transport.[39] When we combine the two MHD instabilities, we find the critical condition for the instability to exist is $\partial\Omega^2/\partial \ln r + g_e \partial \ln T/\partial r > 0$ for weak magnetic field, where $g_e = g - r\Omega^2$.[2] It is easy to see that the two instabilities will excite each other to make the system more unstable if the two factors in the inequality above, i.e., $\partial\Omega^2/\partial \ln r$ and $g_e \partial \ln T/\partial r$ are both negative. On the contrary, if one of them is positive, for instance, $\partial\Omega^2/\partial \ln r > 0$, meaning that the MRI does not come into being in the absence of MTI, this rotational term shows to depress the MTI. The temperature gradient term is needed to be more negative to arouse the instability.

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