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Effects of external magnetic field on propagation of electromagnetic wave in uniform magnetized plasma slabs

Fanya Jin [a,b], Honghui Tong [b], Zhongbing Shi [b], Deli Tang [a,b], Paul K. Chu [a,*]

[a] *Department of Physics and Materials Science, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong*
[b] *Southwestern Institute of Physics, Chengdu, Sichuan 610041, China*

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Abstract

A simple method is proposed to describe the propagation of electromagnetic waves in magnetized uniform plasma slabs. Using this method, the reflection, absorption and transmission coefficients of such plasmas for right-hand circularly waves are studied and the effects of the continuously changing external magnetic field on the power of the electromagnetic waves propagated in magnetized plasma slabs with fixed parameters are presented. Our method enables more detailed numerical analyses which are useful in practical applications pertaining to the control of the reflection or absorption coefficients of electromagnetic wave through a uniform magnetized plasma slab by adjusting the external magnetic field.

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Absorbers and reflectors of electromagnetic waves have many applications, and so the interactions between electromagnetic waves and low-temperature plasma, particularly numerical analysis of reflection, absorption, and transmission of electromagnetic waves in unmagnetized, magnetized, nonuniform, and uniform plasma, have been a topic of intensive research [1–14]. For instance, Laroussi numerically evaluated the interactions between electromagnetic waves and a uniform magnetized plasma cylinder [14], and Helaly and Yeh studied the scattering from nonuniform magnetized plasma cylinders [12,13]. Based on the Fresenel coefficients formulation and ideal effective input impedance, Laroussi and Roth investigated the electromagnetic wave propagation through nonuniform plasma slabs and Ruck et al. analyzed electromagnetic waves reflection from dielectric slabs by using the 2×2 matrix approach [4,15]. By using a scattering matrix method, the reflection, absorption, and transmission from nonuniform magnetized plasma slabs were also studied by Hu and Tang [8,10].

In this paper, a simple model is proposed to describe the behavior of electromagnetic waves in unmagnetized and magnetized uniform plasma slabs. In comparison with previous investigations that mostly focus on plasma physics and theories, our study stresses accurate computation. In addition, our model allows for the minimization of reflection and/or maximization of absorption by continuously varying parameters such as the magnetic field. As a result, the effects of the external magnetic field on the power of the electromagnetic wave propagated in plasma slabs can be more accurately described.

In our numerical analysis, the incident wave is assumed to be a plane wave incident at an orthogonal angle into the slab, and the plasma is cold, weakly ionized, steady-state, uniform, and collisional. The plasma slab exhibits dispersion and obeys the following Maxwell's equations:

$$\nabla \times E = -j\omega\mu_0 H, \tag{1}$$

* Corresponding author. Tel.: +852 27887724; fax: +852 27889549.
E-mail address: paul.chu@cityu.edu.hk (P.K. Chu).

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$$\nabla \times H = (\sigma + j\omega\varepsilon_r\varepsilon_0)E, \tag{2}$$

where $J = \sigma E$. Taking into account the oscillatory fields with a time dependence of $\exp(j\omega t)$, Faraday's and Ampere's laws become:

$$\nabla \times \nabla \times E = \nabla\nabla \bullet E - \nabla^2 E = -\frac{\breve{\varepsilon}_r}{c^2}\frac{\partial^2 E}{\partial t^2}, \tag{3}$$

where $\breve{\varepsilon}_r$ is a complex dielectric constant such that

$$j\omega\breve{\varepsilon}_r\varepsilon_0 \to \sigma + j\omega\varepsilon_r\varepsilon_0. \tag{4}$$

We obtain the generalized wave equation

$$\nabla^2 E = \frac{\breve{\varepsilon}_r}{c^2}\omega^2 E, \tag{5}$$

where $c = (\varepsilon_0\mu_0)^{-1/2}$ is the velocity of light in vacuum. We assume that a wave traveling in the z direction has the phase factor $\exp(j\omega t - \breve{\gamma} z)$. The solution of Eq. (5) is

$$E = E_0 \exp(j\omega t - \breve{\gamma} t). \tag{6}$$

We then obtain a complex propagation coefficient $\breve{\gamma}$

$$\breve{\gamma}^2 = -\breve{\varepsilon}_r\frac{\omega^2}{c^2}, \tag{7}$$

and it is generally expressed as

$$\breve{\gamma} = \alpha + j\beta = j(\breve{\varepsilon}_r)^{1/2}\frac{\omega}{c}, \tag{8}$$

where α is the attenuation coefficient and β is the phase coefficient. Hence, Eq. (6) can also be expressed as

$$E = E_0 e^{-\alpha z} e^{j(\omega t - \beta z)}. \tag{9}$$

We assume that one plane wave propagates perpendicular to the uniform plasma slab in the presence of an additional uniform magnetic field B. The schematic diagram of the wave propagation is depicted in Fig. 1. The incident wave travels through the slab with reflections at the interface and absorption by the plasma. The total incident power, reflected power, transmitted power, and absorbed power are designated as P_i, P_r, P_t, and P_a, respectively. The complex dielectric constant $\breve{\varepsilon}_r$ for a plane wave propagating through a cold plasma slab without and with an additional uniform magnetic field can be obtained. For the unmagnetized plasma,

$$\breve{\varepsilon}_r = \left(1 - \frac{\omega_p^2}{\omega^2 + \nu^2}\right) - j\left(\frac{\omega_p^2 \nu/\omega}{\omega^2 + \nu^2}\right), \tag{10}$$

$$\alpha = \frac{\omega}{c}\left\{-\frac{1}{2}\left(1 - \frac{\omega_p^2}{\omega^2 + \nu^2}\right) + \frac{1}{2}\left[\left(1 - \frac{\omega_p^2}{\omega^2 + \nu^2}\right)^2 + \left(\frac{\omega_p^2}{\omega^2 + \nu^2}\frac{\nu}{\omega}\right)^2\right]^{1/2}\right\}^{1/2}, \tag{11}$$

$$\beta = \frac{\omega}{c}\left\{\frac{1}{2}\left(1 - \frac{\omega_p^2}{\omega^2 + \nu^2}\right) + \frac{1}{2}\left[\left(1 - \frac{\omega_p^2}{\omega^2 + \nu^2}\right)^2 + \left(\frac{\omega_p^2}{\omega^2 + \nu^2}\frac{\nu}{\omega}\right)^2\right]^{1/2}\right\}^{1/2}. \tag{12}$$

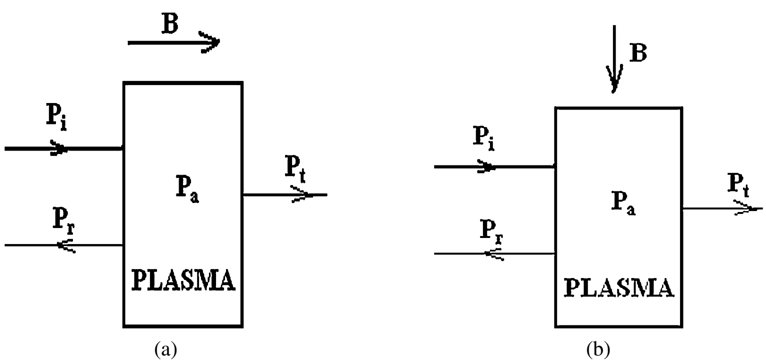

Fig. 1. Schematic diagram of wave propagation: (a) B parallel to the wave propagation direction ($\theta = 0^\circ$), (b) B vertical to the wave propagation direction ($\theta = 90^\circ$).

For the magnetized plasma,

$$\breve{\varepsilon}_r = 1 - \frac{\omega_p^2/\omega^2}{\left[1 - j\frac{\nu}{\omega} - \frac{(\omega_b^2/\omega^2)\sin^2\theta}{2(1-\omega_p^2/\omega^2 - j\nu/\omega)}\right] \pm \left[\frac{(\omega_b^4/\omega^4)\sin^4\theta}{4(1-\omega_p^2/\omega^2 - j\nu/\omega)^2} + \frac{\omega_b^2}{\omega^2}\cos^2\theta\right]^{1/2}}, \tag{13}$$

where $\omega_p^2 = (ne^2)/(m\varepsilon_0)$ is the plasma frequency, ω is the microwave frequency, $\omega_b = (neB)/m$ is the electron gyrofrequency, ν is the effective collision frequency between the electron and neutral gas, θ is the angle of propagation with respect to the static background magnetic field, m is the mass of the electron, and ε_0 is the permittivity in free space. The $\pm$ sign indicates the left- and right-hand polarization wave.

In our model, we initially assume wave propagating along the magnetic field ($\theta = 0°$) and right-hand polarization. The equivalent complex dielectric constants for right-hand polarization wave can be obtained as follows:

$$\breve{\varepsilon}_r = \left\{1 - \frac{\omega_p^2(\omega-\omega_b)}{\omega[(\omega-\omega_b)^2+\nu^2]}\right\} - j\left\{\frac{\omega_p^2\nu}{\omega[(\omega-\omega_b)^2+\nu^2]}\right\} \tag{14}$$

and

$$\alpha = \frac{\omega}{c}\left[-\frac{1}{2}\left\{1 - \frac{\omega_p^2(\omega-\omega_b)}{\omega[(\omega-\omega_b)^2+\nu^2]}\right\} + \frac{1}{2}\left(\left\{1 - \frac{\omega_p^2(\omega-\omega_b)}{\omega[(\omega-\omega_b)^2+\nu^2]}\right\}^2 + \left\{\frac{\omega_p^2\nu}{\omega[(\omega-\omega_b)^2+\nu^2]}\right\}^2\right)^{1/2}\right]^{1/2}, \tag{15}$$

$$\beta = \frac{\omega}{c}\left[\frac{1}{2}\left\{1 - \frac{\omega_p^2(\omega-\omega_b)}{\omega[(\omega-\omega_b)^2+\nu^2]}\right\} + \frac{1}{2}\left(\left\{1 - \frac{\omega_p^2(\omega-\omega_b)}{\omega[(\omega-\omega_b)^2+\nu^2]}\right\}^2 + \left\{\frac{\omega_p^2\nu}{\omega[(\omega-\omega_b)^2+\nu^2]}\right\}^2\right)^{1/2}\right]^{1/2}. \tag{16}$$

We then assume wave propagation across the magnetic field ($\theta = 90°$) and right-hand polarization. The equivalent complex dielectric constant is

$$\breve{\varepsilon}_r = \left\{1 - \frac{\omega_p^2[(\omega^2-\omega_p^2)(\omega^2-\omega_p^2-\omega_b^2)+\nu^2\omega^2]}{\omega^2(\omega^2-\omega_p^2-\omega_b^2-\nu^2)^2+\nu^2(2\omega^2-\omega_p^2)^2}\right\} - j\left\{\frac{\nu\omega_p^2[\omega_p^4+\omega^2(\omega^2-2\omega_p^2+\omega_b^2+\nu^2)]}{\omega[\omega^2(\omega^2-\omega_p^2-\omega_b^2-\nu^2)^2+\nu^2(2\omega^2-\omega_p^2)^2]}\right\} \tag{17}$$

and

$$\begin{aligned}\alpha = \frac{\omega}{c}\Bigg[&\frac{1}{2}\left(\left\{1 - \frac{\omega_p^2[(\omega^2-\omega_p^2)(\omega^2-\omega_p^2-\omega_b^2)+\nu^2\omega^2]}{\omega^2(\omega^2-\omega_p^2-\omega_b^2-\nu^2)^2+\nu^2(2\omega^2-\omega_p^2)^2}\right\}^2 + \left\{\frac{\nu\omega_p^2[\omega_p^4+\omega^2(\omega^2-2\omega_p^2+\omega_b^2+\nu^2)]}{\omega[\omega^2(\omega^2-\omega_p^2-\omega_b^2-\nu^2)^2+\nu^2(2\omega^2-\omega_p^2)^2]}\right\}^2\right)\\ &- \frac{1}{2}\left\{1 - \frac{\nu\omega_p^2[\omega_p^4+\omega^2(\omega^2-2\omega_p^2+\omega_b^2+\nu^2)]}{\omega[\omega^2(\omega^2-\omega_p^2-\omega_b^2-\nu^2)^2+\nu^2(2\omega^2-\omega_p^2)^2]}\right\}\Bigg]^{1/2},\end{aligned} \tag{18}$$

$$\begin{aligned}\beta = \frac{\omega}{c}\Bigg[&\frac{1}{2}\left(\left\{1 - \frac{\omega_p^2[(\omega^2-\omega_p^2)(\omega^2-\omega_p^2-\omega_b^2)+\nu^2\omega^2]}{\omega^2(\omega^2-\omega_p^2-\omega_b^2-\nu^2)^2+\nu^2(2\omega^2-\omega_p^2)^2}\right\}^2 + \left\{\frac{\nu\omega_p^2[\omega_p^4+\omega^2(\omega^2-2\omega_p^2+\omega_b^2+\nu^2)]}{\omega[\omega^2(\omega^2-\omega_p^2-\omega_b^2-\nu^2)^2+\nu^2(2\omega^2-\omega_p^2)^2]}\right\}^2\right)\\ &+ \frac{1}{2}\left\{1 - \frac{\nu\omega_p^2[\omega_p^4+\omega^2(\omega^2-2\omega_p^2+\omega_b^2+\nu^2)]}{\omega[\omega^2(\omega^2-\omega_p^2-\omega_b^2-\nu^2)^2+\nu^2(2\omega^2-\omega_p^2)^2]}\right\}\Bigg]^{1/2}.\end{aligned} \tag{19}$$

For the normal incident wave, the reflection coefficient at the interface between free space and the uniform plasma slab is

$$\Gamma = \frac{1-\sqrt{\breve{\varepsilon}_r}}{1+\sqrt{\breve{\varepsilon}_r}}. \tag{20}$$

The reflection power can be obtained by:

$$P_r = P_i|\Gamma|^2 = P_i\left|\frac{1-\sqrt{\breve{\varepsilon}_r}}{1+\sqrt{\breve{\varepsilon}_r}}\right|^2, \tag{21}$$

and the transmitted power is given by

$$P_t = (P_i - P_r)\exp(-2\alpha d), \tag{22}$$

where d is the width of the plasma slab. Finally, the absorbed power is obtained by subtracting the reflected power and transmitted power from the incident power

$$P_a = P_i - P_r - P_t. \tag{23}$$

In order to avoid calculation of complex numbers, the complex dielectric constant can be described as

$$\breve{\varepsilon}_r = \varepsilon' - j\varepsilon'', \tag{24}$$

where $\varepsilon' = \mathrm{Re}(\breve{\varepsilon}_r)$ and $\varepsilon'' = \mathrm{Im}(\breve{\varepsilon}_r)$. By changing the complex angle, Eq. (24) can be expressed as

$$\breve{\varepsilon}_r = \varepsilon e^{-j\theta_\varepsilon}, \qquad \varepsilon = \sqrt{\varepsilon'^2 + \varepsilon''^2}, \qquad \theta_\varepsilon = \mathrm{arctg}\,\frac{\varepsilon''}{\varepsilon'}. \tag{25}$$

Then Eq. (20) can be expressed as

$$\Gamma = \frac{1-\sqrt{\breve{\varepsilon}_r}}{1+\sqrt{\breve{\varepsilon}_r}} = \frac{1-\sqrt{\varepsilon}e^{-j\frac{\theta_\varepsilon}{2}}}{1+\sqrt{\varepsilon}e^{-j\frac{\theta_\varepsilon}{2}}}. \tag{26}$$

Hence, for $|\Gamma|^2$, we have

$$|\Gamma|^2 = \left|\frac{1-\sqrt{\varepsilon}e^{-j\frac{\theta_\varepsilon}{2}}}{1+\sqrt{\varepsilon}e^{-j\frac{\theta_\varepsilon}{2}}}\right|^2 = \left|\frac{1-\sqrt{\varepsilon}(\cos\frac{\theta_\varepsilon}{2} - j\sin\frac{\theta_\varepsilon}{2})}{1+\sqrt{\varepsilon}(\cos\frac{\theta_\varepsilon}{2} - j\sin\frac{\theta_\varepsilon}{2})}\right|^2 = \frac{(1+\varepsilon\sin^2\frac{\theta_\varepsilon}{2} - \varepsilon\cos^2\frac{\theta_\varepsilon}{2})^2 + 4\varepsilon\sin^2\frac{\theta_\varepsilon}{2}\cos^2\frac{\theta_\varepsilon}{2}}{(1+\sqrt{\varepsilon}\cos\frac{\theta_\varepsilon}{2})^2 + \varepsilon\sin^2\frac{\theta_\varepsilon}{2}}. \tag{27}$$

More details of the calculation of the reflection, absorption and transmission ratios versus wave frequency and magnetic field strength with the external magnetic field B parallel to the wave propagation direction ($\theta = 0°$) and right-hand polarization are provided here. The real and imaginary permittivity part of the plasma can be obtained by Eq. (14):

$$\varepsilon' = 1 - \frac{\omega_p^2(\omega - \omega_b)}{\omega[(\omega-\omega_b)^2 + \nu^2]}, \qquad \varepsilon'' = \frac{\omega_p^2 \nu}{\omega[(\omega-\omega_b)^2 + \nu^2]} \tag{28}$$

and so ε and θ_ε can be expressed as follows:

$$\varepsilon = \sqrt{\left(1 - \frac{\omega_p^2(\omega - \omega_b)}{\omega[(\omega-\omega_b)^2 + \nu^2]}\right)^2 + \left(\frac{\omega_p^2 \nu}{\omega[(\omega-\omega_b)^2 + \nu^2]}\right)^2},$$
$$\theta_\varepsilon = \mathrm{arctg}\,\frac{\omega_p^2\nu/\omega[(\omega-\omega_b)^2+\nu^2]}{1-\omega_p^2(\omega-\omega_b)/\omega[(\omega-\omega_b)^2+\nu^2]} \tag{29}$$

By substituting into Eq. (27), $|\Gamma|^2$ can be obtained. Subsequently, the reflection power, transmitted power and absorbed power are obtained by Eqs. (21)–(23). The same method is used to compute the data when the magnetic field B is vertical to the wave propagation direction ($\theta = 90°$). Before the calculation, the designed plasma parameters such as plasma density, plasma slab width, and the effective collision frequency between the electron and neutral gas are initially fixed. By changing the external magnetic field and frequency of electromagnetic wave continuously, the corresponding reflection, absorption and transmission coefficients of the plasma are obtained using the procedures described above.

In our model, the uniform plasma slab density n is $1 \times 10^{18}\ \mathrm{m}^{-3}$ corresponding to a plasma frequency of about 9 GHz and a width of 10 cm, and the collision frequency ν is 5 GHz. The wave frequency and external magnetic field strength are varied continuously from 1 to 18 GHz and 0 T to 0.5 T, respectively. Double precision is adopted in our algorithm. However, it should be noted that parameters such as the magnetic field are varied continuously and hence, the calculation is linear. The Pentium 4 CPU calculation time is less than 1 s for each simulation. The reflection, absorption, and transmission results are presented as plots of the ratios of the absorption, reflection and transmission power to the incident power versus wave frequency and external magnetic field.

Figs. 2, 3 and 4 show the reflection, absorption and transmission ratios versus wave frequency and magnetic field strength with the external magnetic field B parallel to the wave propagation direction ($\theta = 0\,°$) and right-hand polarization. Figs. 5, 6 and 7 show the data when the magnetic field B is vertical to the wave propagation direction ($\theta = 90°$). It is clear that the resonant absorption band moves from low to high frequencies when the magnetic field strength increases. Accordingly, the reflection ratios decrease substantially with increasing external field strength for $\theta = 0°$, but there are no obvious changes for $\theta = 90°$. The reason is believed to be that high hybrid frequency resonant absorption occurs in the magnetized plasmas, which correlates to the plasma frequency and gyrofrequency. The external magnetic field strength brings changes to the gyrofrequency thereby affecting the high-resonant absorption frequency. When the electromagnetic wave with right-hand polarization (called extraordinary wave) propagates across the magnetic field ($\theta = 90°$), the characteristic waves are obtained by:

$$\breve{\varepsilon}_r = 1 - \frac{\omega_p^2/\omega^2}{1 - j\frac{\nu}{\omega} - (\omega_b^2/\omega^2)/(1-\omega_p^2/\omega^2 - j\nu/\omega)}, \tag{30}$$

where $\omega_p^2 = (ne^2)/(m\varepsilon_0)$ is the plasma frequency, ω is the microwave frequency, $\omega_b = (neB)/m$ is the electron gyrofrequency, ν is the effective collision frequency between the electron and neutral gas. The dispersion relation ($\omega(\kappa)$) of extraordinary wave can be expressed by the wave number (κ) in noncollisional ($\nu = 0$) magnetic plasma as:

$$\frac{c^2\kappa^2}{\omega^2} = 1 - \frac{\omega_p^2(\omega^2 - \omega_p^2)}{\omega^2(\omega^2 - \omega_h^2)}, \tag{31}$$

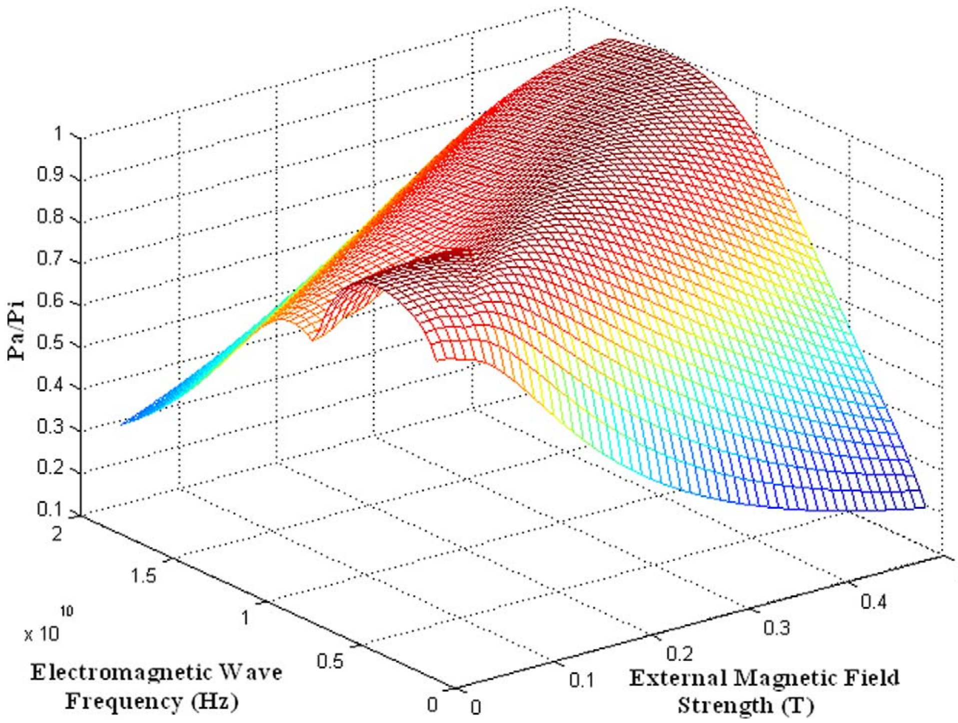

Fig. 2. Absorption ratios versus wave frequency and magnetic field strength for $n = 1 \times 10^{18}\ \mathrm{m}^{-3}$, $\nu = 5$ GHz, $d = 10$ cm, $\theta = 0°$, and right-hand polarization.

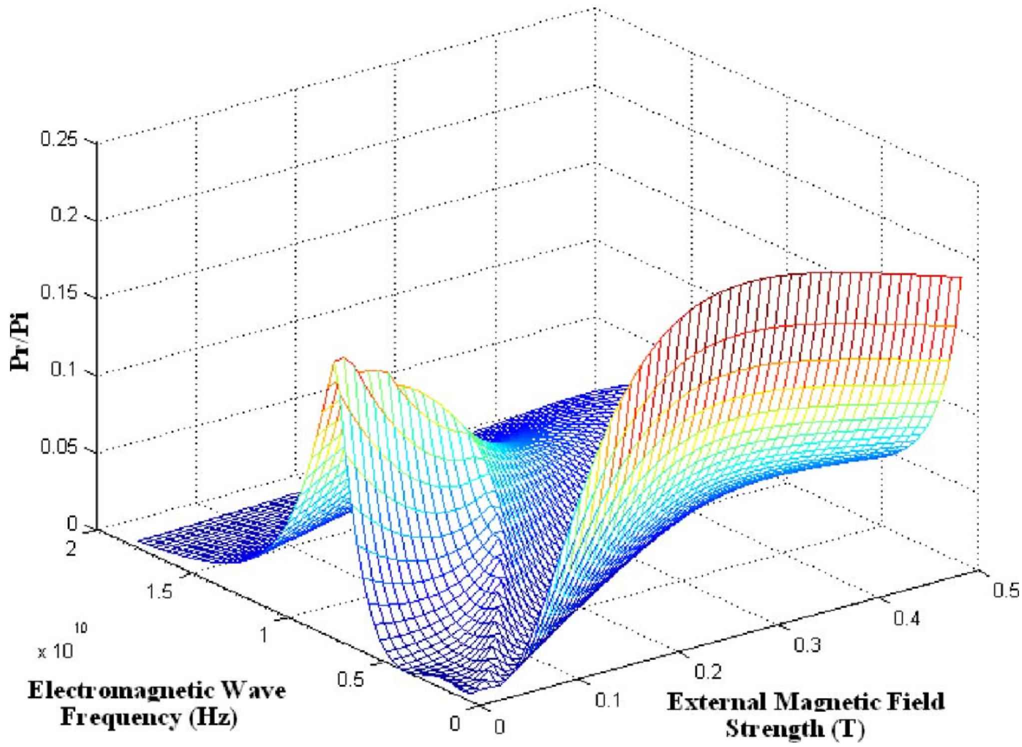

Fig. 3. Reflection ratios versus wave frequency and magnetic field strength for $n = 1 \times 10^{18}\ \mathrm{m}^{-3}$, $\nu = 5$ GHz, $d = 10$ cm, $\theta = 0°$, and right-hand polarization.

where $\omega_h = \sqrt{\omega_{ce}^2 + \omega_p^2}$ is called high hybrid frequency. When $\omega \to \omega_h$, $\kappa \to \infty$. This also shows that the wave length will approach to 0 and the resonance effect occurs when the frequency of the electromagnetic wave approaches the high hybrid frequency. Under this condition, the absorption of electromagnetic wave by the plasma will reach the maximum value, and this kind of absorption is called high hybrid frequency resonant absorption.

In summary, a simple method which avoids calculation of complex numbers is presented here. The model can be used to numerically investigate the reflection, absorption and transmission of electromagnetic waves through a unmagnetized and

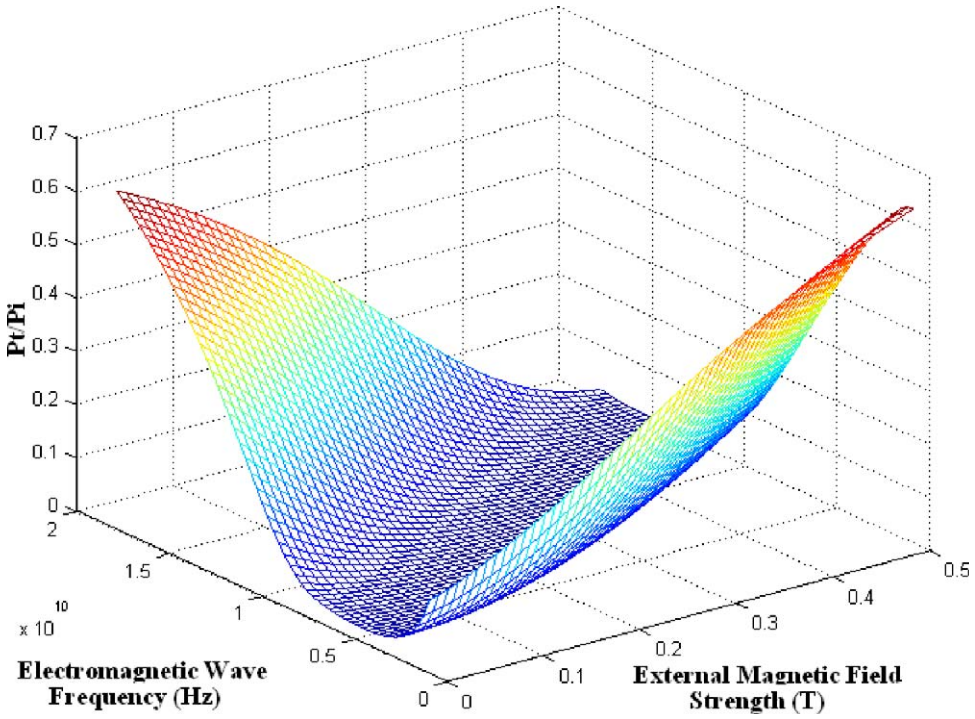

Fig. 4. Transmission ratios versus wave frequency and magnetic field strength for $n = 1 \times 10^{18}\ \mathrm{m}^{-3}$, $\nu = 5$ GHz, $d = 10$ cm, $\theta = 0°$, and right-hand polarization.

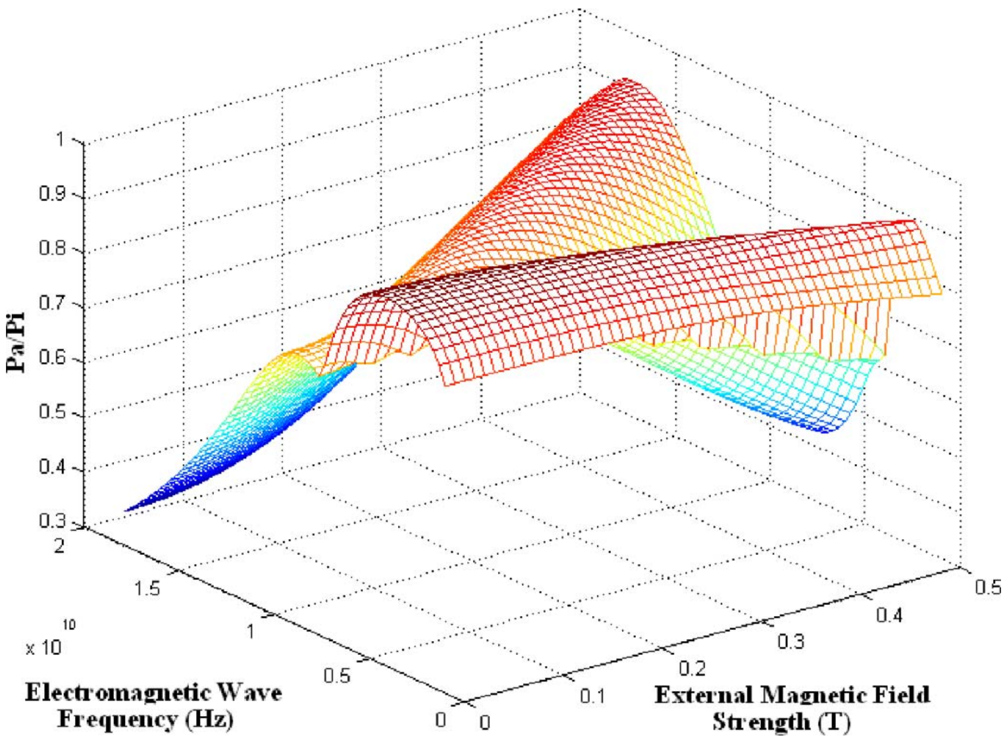

Fig. 5. Absorption ratios versus wave frequency and magnetic field strength for $n = 1 \times 10^{18}\ \mathrm{m}^{-3}$, $\nu = 5$ GHz, $d = 10$ cm, $\theta = 90°$, and right-hand polarization.

magnetized uniform plasma slab. Different from previous studies, we investigate the effects in the presence of continuously varying external magnetic field strength. Consequently, more detailed and accurate results can be obtained. With regard to selective broadband absorption and reflection of electromagnetic waves, the effects induced by varying one parameter such as the external magnetic field can be obtained easily for a magnetized uniform plasmas slab when the other parameters are fixed.

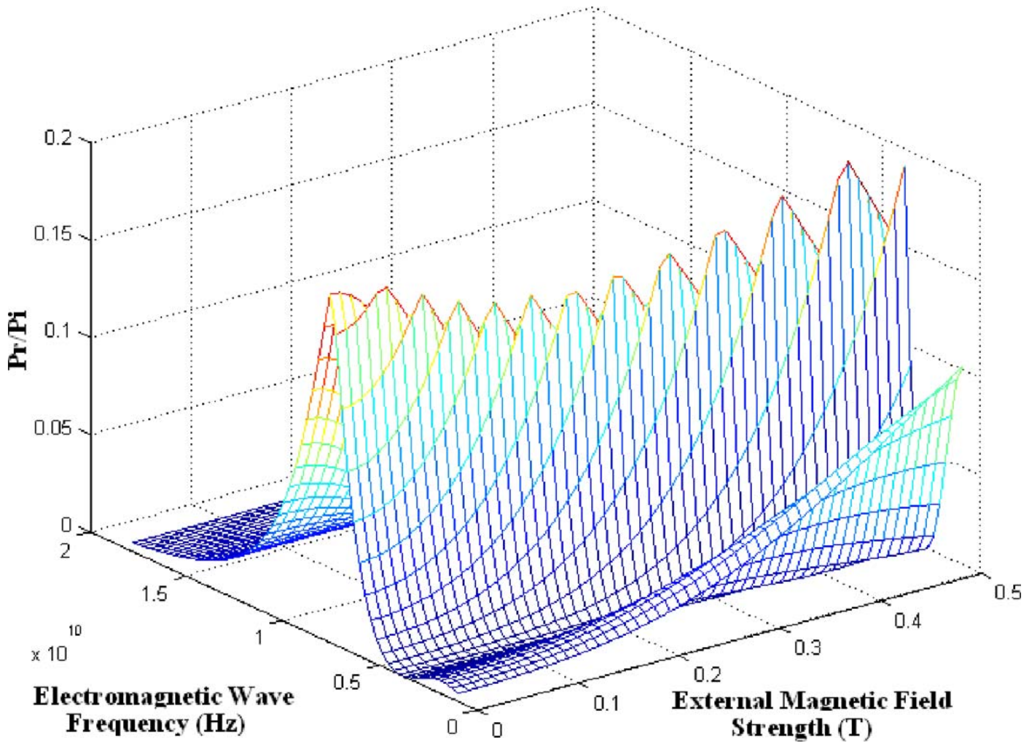

Fig. 6. Reflection ratios versus wave frequency and magnetic field strength for $n = 1 \times 10^{18}\ \mathrm{m}^{-3}$, $\nu = 5$ GHz, $d = 10$ cm, $\theta = 90°$, and right-hand polarization.

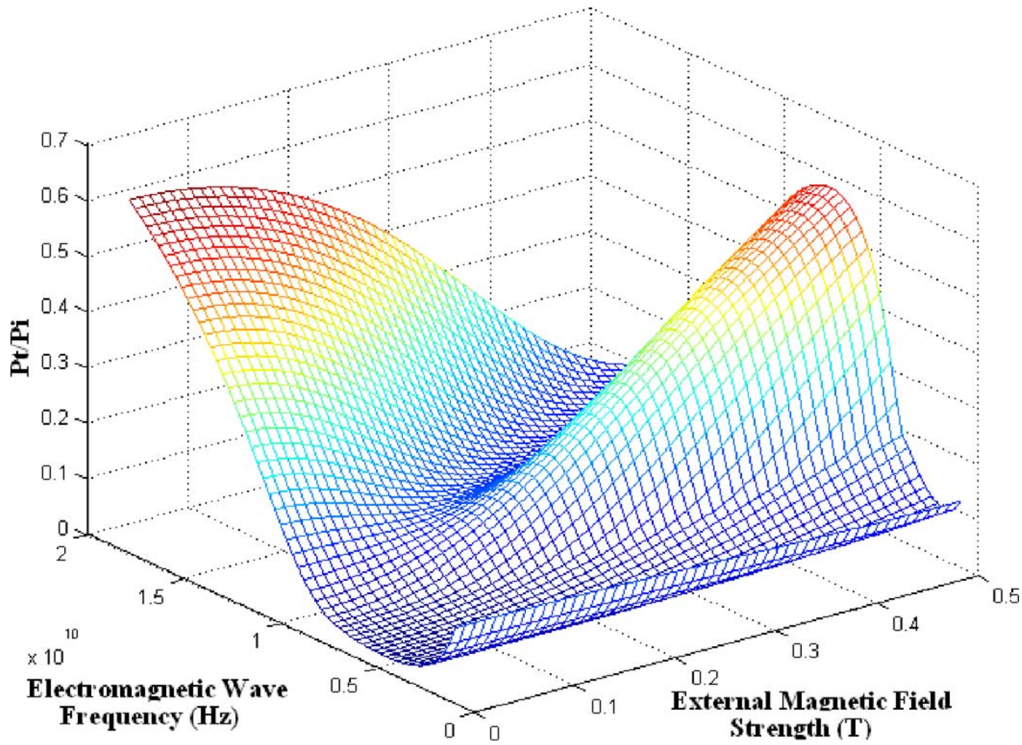

Fig. 7. Transmission ratios versus wave frequency and magnetic field strength for $n = 1 \times 10^{18}\ \mathrm{m}^{-3}$, $\nu = 5$ GHz, $d = 10$ cm, $\theta = 90°$, and right-hand polarization.

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