Quadratic Differentials for Laguerre Polynomials with Complex Coefficients

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The definition of the classical Laguerre polynomials $L_n^{(\alpha)}$ makes sense when the parameter $\alpha$ takes non-classical real (i.e., $\alpha \leq -1$) or even complex values. In these cases the zeros of $L_n^{(\alpha)}$ depart from the positive semi-axis and distribute along certain curves on the complex plane. These curves, that become apparent in the asymptotic regime

$$p_n(z) = L_n^{(\alpha_n)}(nz), \quad \text{with}\lim_{n} \frac{\alpha_n}{n} = A \in \mathbb{C},$$

are trajectories of the quadratic differential

$$\omega_A = \frac{4z - (z - A)^2}{z^2} \, dz^2,$$

on the Riemann sphere $\overline{\mathbb{C}}$. They exhibit also the $S$-property of H. Stahl, which characterizes the support of the limiting zero-counting measure of $p_n$’s.

For $A \in \mathbb{R}$, these trajectories have been described in [1, 2, 3]. In this talk we discuss the global structure of both the trajectories and the orthogonal trajectories of $\omega_A$ when $A \notin \mathbb{R}$. This information allows to apply, in the spirit of [1, 2] the non-linear steepest descent asymptotic analysis of Deift and Zhou to the Riemann-Hilbert characterization of $p_n$’s in order to establish their strong asymptotics on the whole complex plane.

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References

