A Sparse Dual Frame Approach to Compressed Sensing with General Frames

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Compressed sensing with general frames D as sparse dictionaries is derived from a large number of applications in signal processing. In these applications, $f = Dx$ and x is sparse. The measurement of f is given by $y = Af = ADx$. The ℓ_1-synthesis approach is to recover f by first finding the sparse coefficient x via

$$\hat{x} = \underset{x}{\operatorname{argmin}} \|x\|_1 \quad s.t. \ \|y - ADx\|_2 \leq \epsilon,$$

where ϵ is the noise bound, and then derive f by the synthesis operation $\hat{f} = D\hat{x}$. An alternative way is to find f directly from

$$\hat{f} = \underset{f}{\operatorname{argmin}} \|D^* f\|_1 \quad s.t. \ \|y - Af\|_2 \leq \epsilon, \tag{1}$$

which is known as the ℓ_1-analysis approach. Here the analysis operator D^* stands for the complex conjugate transpose of D. There are a number of studies of both schemes. None of the two approaches is generally satisfactory.

We present an approach where the analysis operator is the sparse dual frame $\tilde{D}$ of D so that $\tilde{D}^* f$ is (truly) sparse. An ℓ_1-analysis procedure based on such a sparse analysis operator $\tilde{D}^*$ is seen in examples capable of recovering f when both ℓ_1-synthesis and the ℓ_1-analysis of (1) fail by a large margin. Evidently, the determination of the sparse dual $\tilde{D}$ is an ill-posted inverse problem as well. A notion of sparse duals and an iterative procedure will be introduced. Examples, some analysis of the algorithm and a sufficient recovery condition by the sparse analysis operator $\tilde{D}$ will be presented. This is a preliminary report, and a joint work with Yulong Liu and Tiebin Mi.