Title: Hyperbolic Resonant Systems of Balance Laws
Speaker: Professor Meng-Kai HONG, Department of Mathematics, National Central University

Abstract: In this talk, we consider some hyperbolic systems of balance laws whose eigenvalues of the Jacobian matrix coincide in some transition curves (sonic states). I will first review some results of global weak solutions established by Isaacson-Temple and Hong-Temple. The conditions for generic Riemann solutions are given. The Glimm functionals defined in singular (z,w) coordinate were introduced by Hong-Temple for the stability of generalized Glimm scheme. We extend their results to the hyperbolic waves of generalized Keller-Segel equations.

Title: Bound States in the Continuum on Periodic Structures: Theory and Applications
Speaker: Professor Ya Yan LU, Department of Mathematics, City University of Hong Kong

Abstract: It is well-known that boundary value problems for the scattering of plane incident waves by periodic structures may not have uniqueness at some values of the frequencies, and such a loss of uniqueness is associated with guided modes in the radiation continuum (i.e., above the lightline), also called bound states in the continuum (BICs), and mathematically, discrete eigenvalues in a continuous spectrum. In this talk, we present recent numerical and perturbation results on BICs for a 2D periodic structures, discuss some applications related to the discontinuities of reflection/transmission coefficients, total reflection/transmission phenomenon, arbitrarily strong resonances, and nonuniqueness of nonlinear diffraction problems. This is a joint work with Lijun Yuan of Chongqing Technology and Business University and Zhen Hu of Hohai University.

Title: A Simple Direct-forcing Immersed Boundary Projection Method with Prediction-correction for Fluid-solid Interaction Problems
Speaker: Professor Suh-Yuh YANG, Department of Mathematics, National Central University

Abstract: In this work, we will introduce a simple and novel prediction-correction approach in conjunction with a direct-forcing immersed boundary projection method for simulating the dynamics of fluid-solid interaction problems, where the immersed solid object can be stationary or moving in the fluid with a prescribed velocity. The main idea of this approach is based on the introduction of a virtual force of discrete type, which is distributed on the whole solid body and appended to the fluid momentum equations to accommodate the no-slip boundary condition at the immersed solid boundary. More specifically, based on the rate of moment changes of the solid body, we first predict the virtual force at the grid points by using the difference between the prescribed solid velocities and the computed velocities which are obtained by applying the projection method to solve the incompressible Navier-Stokes equations on the whole domain in the absence of immersed solid object. This predicted virtual force is then added to the fluid momentum equations as an additional
forcing term and again, we employ the projection method to correct the velocity field, pressure and virtual force. Such prediction-correction procedure can be iterated to generate a more general method, if necessary. Numerical experiments of several benchmark problems are performed to illustrate the simplicity and high performance of the prediction-correction approach. We find that our numerical results are in very good agreement with the previous works in the literature and in most cases, one correction step is good enough. This is joint work with Tzyy-Leng Horng, Po-Wen Hsieh and Cheng-Shu You.

**Title:** Small Noise Expansions and Fast Numerical Methods for Elliptic Equation on Domains with Random Boundary  
**Speaker:** Dr Xiang ZHOU, Department of Mathematics, City University of Hong Kong

**Abstract:** We consider the solution to an elliptic partial differential equation on the domain whose geometric boundary is subject to certain small perturbations, including the interface problem where the diffusion tensor may have a jump across the boundary. As the perturbation amplitude tends to zero, we propose an asymptotic approach to construct, up to any order in theory, the approximate series of the true solution on the random domain as the sum of the solutions on the unperturbed regular domain with random boundary conditions. Our derivation is simple and intuitive by the Taylor expansion of the solution on the boundary, but it completely recovers the existing equivalent results from the multiscale expansion and the shape calculus. For the interface problem, when the ratio of diffusion coefficients across the interface is large, a two-scale asymptotic expansion are derived. An example from measuring photoluminescence in organic semiconductor is provided to show the numerical efficiency of our method.

**Title:** Convexity of Transonic Shock in the Self-similar Coordinates and Applications  
**Speaker:** Dr Wei XIANG, Department of Mathematics, City University of Hong Kong

**Abstract:** Convexity of shocks is frequently observed in many experimental results and provides better understanding of mathematical problems with the nonlinear wave, the uniqueness of instance. We consider the pseudo-transonic shock governed by the potential flow equation in the self-similar coordinates, and give a framework to show the strict and uniform convexity by a nonlinear and global argument. Finally, several applications are given. This work is collaborated with Prof. G.Q. Chen and Prof. Feldman.
Title: Potentially Singular Solutions of the 3D Axisymmetric Euler Equations  
Speaker: Dr Guo LUO, Department of Mathematics, City University of Hong Kong

Abstract: Whether the 3D incompressible Euler equations can develop a singularity in finite time from smooth initial data is one of the most challenging problems in mathematical fluid dynamics. This work attempts to provide an affirmative answer to this long-standing open question from a numerical point of view, by presenting a class of potentially singular solutions to the Euler equations computed in axisymmetric geometries. The solutions satisfy a periodic boundary condition along the axial direction and no-flow boundary condition on the solid wall. The equations are discretized in space using a hybrid 6th-order Galerkin and 6th-order finite difference method, on specially designed adaptive (moving) meshes that are dynamically adjusted to the evolving solutions. With a maximum effective resolution of over $(3*10^{12})^2$ near the point of the singularity, we are able to advance the solution up to $\tau_2 = 0.003505$ and predict a singularity time of $t_s \approx 0.0035056$, while achieving a pointwise relative error of $O(10^{-4})$ in the vorticity vector and observing a $(3*10^8)$-fold increase in the maximum vorticity. The numerical data is checked against all major blowup (non-blowup) criteria, including Beale-Kato-Majda, Constantin-Fefferman-Majda, and Deng-Hou-Yu, to confirm the validity of the singularity. A local analysis near the point of the singularity also suggests the existence of a self-similar blowup. We also discuss a 1D model which can be viewed as a local approximation to the Euler equations near the solid boundary of the cylinder. The finite-time blowup of this 1D model is proved for a class of smooth initial data, which are essentially restrictions of the initial data used in the full 3D blowup calculations.

Title: An Inextensible Vesicle Interacting with Navier-Stokes Equations  
Speaker: Professor Ching-Hsiao CHENG, Department of Mathematics, National Central University

Abstract: In this talk, we study the dynamics of inextensible vesicles inside viscous Newtonian fluids. I will first talk about how to derive a mathematical model describing the underlying physics and then talk about the a priori estimates in order to capture the possible solution spaces in which we look for possible solutions. Some new information which may enhance the accuracy in numerical simulations will be provided.

Title: Asymptotic Limit for Rotational Compressible Magnetohydrodynamic Flows  
Speaker: Ms Cheng-Fang SU, Department of Mathematics, National Central University

Abstract: In this talk, we consider the compressible models of magnetohydrodynamic flows giving rise to a variety of mathematical problems in many areas. First, we will introduce the asymptotic limit for the compressible rotational magnetohydrodynamic flows with the well-prepared initial data such that a rigorous quasi-geostrophic equation with diffusion term governed by the magnetic field from a compressible rotational magnetohydrodynamic flows is derived. Finally, we will show the two results: the existence of the unique global strong solution of quasi-geostropic equation with good regularity on the velocity and magnetic field and the derivation of quasi-geostropic equation with diffusion.
Title: Convex Splitting Method for the Calculation of Transition States of Energy Functional  
Speaker: Ms Shuting GU, Department of Mathematics, City University of Hong Kong

Abstract: Among numerical methods for partial differential equations arising from steepest descent dynamics of energy functionals (e.g., Allen-Cahn and Cahn-Hilliard equations), the convex splitting method is well-known to maintain unconditional energy stability for a large time step size. In this work, we show how to use the convex splitting idea to solve the problem of finding transition states, i.e., index-1 saddle points of the same energy functionals. Based on the previous work of iterative minimization formulation (IMF) for saddle points (SIAM J. Numer. Anal., vol. 53, p1786, 2015), we introduce the convex splitting idea to minimize the auxiliary functional at each cycle of the IMF. We present a general principle of constructing convex splitting forms for these auxiliary functionals and show how to avoid solving nonlinear equations. The new numerical scheme based on the convex splitting method allows for a much larger time step size than a traditional semi-implicit scheme we tested and it turns out that a large time step has a huge advantage in computational efficiency. The new methods are tested numerically for the one dimensional Ginzburg-Landau energy functional in the search of the Allen-Cahn or Cahn-Hilliard types of transition states.

Title: Local-in-time Well-posedness Theory for MHD Boundary Layer in Sobolev Spaces without Monotonicity  
Speaker: Dr Chengjie LIU, Department of Mathematics, City University of Hong Kong

Abstract: We study the well-posedness theory for MHD boundary layer. Such a kind of boundary layer equations are governed by the Prandtl type equations, which are derived from MHD with no-slip boundary condition for velocity and perfectly conducting conditions for magnetic fields. Under the assumptions that the initial tangential magnetic is always non-zero, we can establish the local-in-time existence, uniqueness, and stability of solutions for the nonlinear MHD boundary layer equations. Compared with the well-posedness theory of classical Prandtl equations, the monotone conditions for the tangential velocity plays a crucial role there. To our surprise, the monotone conditions for the tangential velocity are not needed any more. Precisely, even if the initial velocity admits the non-degenerate critical point, we still can prove the local-in-time well-posedness of the solutions to the nonlinear MHD boundary layer equations in Sobolev spaces, provided that the initial tangential magnetic field is non-zero. Our results improve the classical local well-posedness results of classical Prandtl equations. At the same time our results are in agreement with the general physical cognition that the magnetic field has a stabilizing effect in MHD boundary layer.

Title: On Stability of the 3X3 Fanno-Rayleigh Equation  
Speaker: Dr Shih-Wei CHOU, Department of Mathematics, National Central University

Abstract: In this talk, we study the L1 stability for compressible Euler equations with friction and heat effect that modeled the Fanno-Rayleigh flows through the nozzle with symmetric variable cross-section area. Under suitable assumptions on source, we present the global existence of transonic entropy solutions. By employing a nonlinear functional and estimate the nonlinear wave interactions, we show that that the stability of the L1 norm of the initial perturbation.
**Title:** Dissipation in Parabolic SPDEs  
**Speaker:** Professor Shang-Yuan SHIU, Department of Mathematics, National Central University  

**Abstract:** We consider the following stochastic heat equation (SHE) 
\[
\frac{\partial}{\partial t} u(t,x) = \Delta u(t,x) + \lambda \sigma (u(t,x)) \frac{\partial^2}{\partial t \partial x} \epsilon(t,x), \quad x \in [-1,1]
\]  
with the periodic boundary condition and the initial data is a constant. A paper by Kim and Khoshnevisan and later on a paper by Foondun and Joseph together show that for fixed time \( t \), the second moment of the solution \( u(t,x) \) grows like \( \exp (\lambda^4 t) \) as \( \lambda \) goes to \( \infty \). However, when we consider \( \sup_{x \in [0,1]} u(t,x) \), it converges to 0 in probability as \( \lambda \) goes to \( \infty \). Similarly, when \( \lambda \) is fixed, the second moment of the solution grows exponentially in \( t \). In this case we still have a dissipation theorem: for a fixed \( \lambda \), \( \sup_{x \in [-1,1]} u(t,x) \) converges to 0 a.s. when \( t \) goes to \( \infty \). The exponential growth of the second moment and two dissipation theorems really say that the solution is intermittent. This is an ongoing work with Kunwoo Kim, Davar Khoshnevisan and Carl Mueller.

**Title:** Stability of Boundary Layer Solutions of Poisson-Nernst-Planck Systems  
**Speaker:** Dr Chia-Yu HSIEH, Department of Mathematics, City University of Hong Kong  

**Abstract:** The Poisson-Nernst-Planck (PNP) system has been widely used to describe the ion transport. In order to see Debye layers, which occur in ionic liquids near electrodes, boundary layer solutions need to be investigated. If the Robin boundary condition is imposed for the electrostatic potential, the PNP system admits a boundary layer solution as a steady state. Radial solutions of PNP equations with variable dielectric coefficients are investigated. By transforming the perturbed problem into another parabolic system with a new energy law, we prove that the \( H^{-1} \)-norm of the solution of the perturbed problem decays exponentially.

**Title:** Analysis and Numerical Tool for Stochastic Morphogen-mediated Patterning System  
**Speaker:** Dr Wing Cheong LO, Department of Mathematics, City University of Hong Kong  

**Abstract:** The patterning of many developing tissues is organized by morphogens. Genetic and environmental perturbations of gene expression, protein synthesis and ligand binding are among the sources of unreliability that limit the accuracy and precision of morphogen-mediated patterning. In the talk, we will discuss the interplay between the robustness of patterning to the changes in receptor synthesis and morphogen synthesis and to the effects of cell-to-cell variability. Our analysis elucidates the trade-offs and constraints that arise as a result of achieving these three performance objectives simultaneously in the context of simple, steady-state morphogen gradients formed by diffusion and receptor-mediated uptake. Also, we will present a new hybrid stochastic numerical method for studying stochastic morphogen system. Applications two nonlinear systems of morphogens demonstrate the effectiveness and benefits of the new hybrid method.
Title: A Full-Space Quasi Lagrange-Newton-Krylov Algorithm for Trajectory Optimization Problems
Speaker: Professor Feng-Nan HWANG, Department of Mathematics, National Central University

Abstract: The objectives of this work are to develop and study the full-space quasi Lagrange-Newton-Krylov (FQLNK) algorithm for solving trajectory optimization problems arising from aerospace industrial applications. As its name suggests, we first convert the constrained optimization problem into an unconstrained one by introducing the augmented Lagrangian parameters. The next step is to find the optimal candidate solution by solving the Karush-Kuhn-Tucker (KKT) condition with the Newton-Krylov method. To reduce the computational cost of constructing the KKT system and meanwhile maintain Newton’s fast convergence we employ the Broyden-Fletcher-Goldfarb-Shanno (BFGS) formula to build an approximation of the (1,1) subblock of the KKT matrix, which is the most expensive part of the overall computation. The BFGS-based FQLNK algorithm exhibits a superior speedup compared to some of the alternatives. We demonstrate our FQLNK algorithm to be a practical approach for designing an optimal trajectory of the launch vehicle in the space missions.

Title: Convergence to Barenblatt solutions for physical vacuum of compressible flows with damping
Speaker: Professor Tao LUO, Department of Mathematics, City University of Hong Kong

Abstract: In this talk, I will present some results joint with Huihui Zeng on the global regularity and long time convergence to Barenblatt solutions for physical vacuum free boundaries of compressible flows with damping.
Title: Local Analysis of HDG Methods for Convection-dominated Diffusion Problems  
Speaker: Mr. Yanyi JIN, Department of Mathematics, City University of Hong Kong

Abstract: In this paper, we establish local error analysis of hybridizable discontinuous Galerkin (HDG) methods for convection-dominated diffusion equation in two types of subdomains away from the layers. The first subdomain is of $O(\log(1/h)h^{1/2})$ away from the interior layers and of $O(\log(1/h)h)$ away from the boundary layers and the second one is of $O(\epsilon)$ away from the outflow of the boundary. Our local error bound for the first subdomain can be improved by a factor of $\log(1/h)$. We use weighted estimates to prove the first result by constructing new weight functions and the inf-sup condition of the bilinear form to prove the second one.

Title: Hyperbolic Quenching Problem with Damping in the MEMS Device  
Speaker: Dr Po-Chih HUANG, Department of Mathematics, National Central University

Abstract: Typically, the device of a microelectromechanical system (MEMS) consists of an electric membrane hanged above a rigid ground plate, connected in series with fixed voltage source and a fixed capacitor. As the potential difference applied between the membrane and the plate causes the membrane to be deformed. In this talk, we provide some criteria for quenching and global existence of the solution, and introduce some unsolved problem relating to this model.

Title: Existence and Instability of Traveling Pulses of Generalized Keller-Segel Equations with Nonlinear Chemical Gradients and Small Diffusions  
Speaker: Mr. Yu-Shuo CHEN, Department of Mathematics, National Central University

Abstract: In this paper, we consider a generalized model of 2 2 Keller-Segel system with nonlinear chemical gradient and small cell diffusion. The existence of the traveling pulses for such equations is established by the methods of geometric singular perturbation (GSP in short) and trapping regions from dynamical systems theory. By the technique of GSP, we show that the necessary condition for the existence of traveling pulses is that their limiting profiles with vanishing diffusion can only consist of the slow flows on the critical manifold of the corresponding algebraic-differential system.

Title: On a Stochastic Shallow Water Wave Equation: Global Existence and Wave Breaking  
Speaker: Mr. Hao TANG, Department of Mathematics, City University of Hong Kong

Abstract: In this talk I will present some existence and wave-breaking results of a stochastic shallow water wave equation. In the deterministic case, this shallow water wave equation is connected with the following two separately integrable soliton equations, the KdV equation and the Camassa–Holm equation. Consider the periodic boundary value problem for this model, we establish the local existence of the martingale solution and the pathwise solution in Sobolev spaces $H_s$ with $s > 4$. Then for the linear noise case, conditions that lead to the global existence of the solution and the wave-breaking phenomena, and their associated probabilities, are also acquired. Finally, we study the pathwise dissipative effect of the linear noise on the traveling waves to the deterministic equations.
**Title:** Martin Boundary for Brownian Motion and Some Applications  
**Speaker:** Mr. Wen-Da CHEN, Department of Mathematics, National Central University

**Abstract:** In this note, we study the Martin boundary for some Gaussian diffusion processes $X_t$. General theory of Martin boundary for Markov process has been well developed in the literature. See Kunita-Watanabe (1965), Dynkin (1969), Salminen (1981). One of its important applications is to give the unique representation of harmonic functions in terms of minimal Martin functions. There are very few concrete examples that properties of Martin boundary are studied, such as Brownian motion. Martin Bounday for 2D Gaussian transient diffusion process were studied in Cranston-Orey-Rosler (1983). They studied the space time Martin boundary for such diffusion process. As a result, they obtained the representation for positive harmonic functions. The space-time Martin boundary is to consider the Martin boundary for $X_t = (t, X_t) \in (0, \infty) \times \mathbb{R}^d$ as Markov process. In this note, we show how to generalize this idea to the high dimensional space. We will discuss the convergence of some h-diffusion process, the diffusion process under h-transform, where $h$ is a positive space time harmonic function. We also consider the relation between a positive space time harmonic function and limiting distribution of h-diffusion process. Finally, we will use the representation of the space-time harmonic function to get the Boundary-Crossing probability.