



Three-dimensional analytical determination of the track parameters: over-etched tracks

D. Nikezic¹, K.N. Yu*

Department of Physics and Materials Science, City University of Hong Kong, Tat Chee Avenue, Kowloon Tong, Kowloon, Hong Kong

Received 10 September 2001; received in revised form 17 September 2002; accepted 25 September 2002

Abstract

Three-dimensional analytical determinations of track parameters are extended to cases where the tracks are in the rounded and spherical phases of development. The equation for the track wall in three dimensions and the equation of contour line of the opening were derived for all types of tracks. The expression for the surface area of the track opening has also been found. The equations come up to the well-known expressions for minor and major axes for the special case of constant track etch rate. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Solid state nuclear detectors; Track parameters; Track etch rate

1. Introduction

The problem of track development has attracted much attention for a long time (e.g., Henke and Benton, 1971; Paretzke et al., 1973; Somogyi and Szalay, 1973; Somogyi, 1980; Fromm et al., 1988; Hatzialekou et al., 1988; Ditlov, 1995; Meyer et al., 1995; Nikezic and Kostic, 1997). Recently, a method for calculating the track parameters based on analytical and three-dimensional consideration was presented (Nikezic, 2000). Consideration was restricted to tracks in the first phase of development, where etching does not reach the end point of the particle range and the track is conical in shape. In the present paper, consideration is extended to the tracks for which etching has passed the end point of the particle track. These over-etched tracks are rounded or spherical in shape.

2. The equation of the track wall

The equation of the track wall can be derived in the following way. Referring to Fig. 1, point *A* on the track wall with coordinates (*x*, *y*) was formed from the point *x*₀ on the particle track. From the point (0, 0), the etching travels with the track etch rate *V*_t along the *x*-axis (which is the particle trajectory) and reaches the point *x*₀ at the time *t*₀. From *x*₀, the etching progresses to point *A* with the bulk etch rate *V*_b. The angle $\delta = \delta(x_0)$ is the angle between *V*_t and *V*_b at point *x*₀ as shown in Fig. 1, and can be found as

$$\delta(x_0) = a \sin\left(\frac{1}{V(x_0)}\right), \quad (1)$$

where $V = V_t(x_0)/V_b$.

From the geometrical consideration, it is clear that

$$y'(x) = -\tan \delta(x_0) = -\frac{1}{\sqrt{V^2(x_0) - 1}}. \quad (2)$$

This equation cannot be used as the track wall equation because the expression on the right depends on *x*₀ while the expression on the left depends on *x*. Noting that *x*₀ = *x* - Δ*x*, we have

$$\Delta x = y(x) \tan \delta(x_0) = -y(x)y'(x) \quad (3)$$

* Corresponding author. Tel.: +852-2788-7812; fax: +852-2788-7830.

E-mail address: peter.yu@cityu.edu.hk (K.N. Yu).

¹ On leave from Faculty of Sciences, University of Kragujevac, 34000 Kragujevac, Yugoslavia.

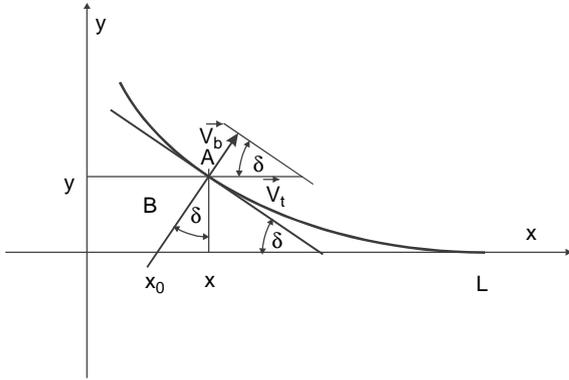


Fig. 1. Geometry of the track wall in two dimensions.

so we can obtain

$$x_0 = x + y(x)y'(x) \quad (4)$$

and

$$y' = -\frac{1}{\sqrt{V^2(x + yy') - 1}}. \quad (5)$$

This is the equation of the track wall in the differential form with both sides depending only on x . Unfortunately this equation cannot be solved analytically. If the angle δ is small or if $V(x)$ is a slowly varying function (which is usually the case in many applications), yy' in the denominator of Eq. (5) can be neglected and the approximation equation of the track wall becomes

$$y = \int_x^L \frac{dx}{\sqrt{V^2(x) - 1}}. \quad (6)$$

This approximation equation was previously used by Nikezic (2000) for the analytical three-dimensional determination of track parameters.

The coordinates (x, y) of point A can be calculated in a simpler way, i.e.,

$$y = B \cos \delta(x_0) \quad (7a)$$

and

$$x = x_0 + B \sin \delta(x_0), \quad (7b)$$

where

$$B = V_b(T - t_0) \quad (7c)$$

and T is total etching time. By using Eqs. (7a)–(7c), the coordinates of the points on the track wall can be generated. A best fit will give

$$y \approx F(x, L) \quad (7d)$$

as the equation of the wall, where L is the distance penetrated by the etching solution (see Fig. 1). However, the information about $V_t(x)$ is lost in this way. Another

possibility is to solve Eq. (5) numerically, but this can be more complicated.

In the special case where $V_t/V_b = V = \text{constant}$, the track wall is represented by a line in two dimensions, and Eq. (7) becomes

$$y_{\text{linear}} = F(x, L) = \frac{-x + L}{\sqrt{V^2 - 1}}. \quad (7e)$$

The equation of the track wall in the conical phase in three dimensions for normal incidence can be written as

$$\sqrt{x^2 + y^2} = F(z, L), \quad (8)$$

where the z -axis is along the particle trajectory, and (x, y) are coordinates of the points in the track wall. The track opening is circular in shape when incidence is normal, but some egg-like shape or droplet-like shape when the incidence is oblique. The contour equation for the opening is given by

$$\sqrt{x'^2 + y'^2} \sin^2 \theta = F(y'' \cos \theta + h/\sin \theta, L), \quad (9)$$

where (x'', y'') are coordinates on the contour of the track opening, θ is the incident angle with respect to the detector surface and h is the total removed layer.

In the cases where V is not a constant, the track opening is not an ellipse, but is instead egg-like, or has even more complicated shapes depending on the functions F and ultimately on the function V .

3. Over-etched tracks: normal incidence

In this section, over-etched tracks will be considered. The schematic sketch of an over-etched track in two dimensions is shown in Fig. 2. After a certain time of etching, the etchant will reach the end point E of the particle range. At that time, the wall of the track is formed and denoted by the number 1 in Fig. 2. Point A with coordinates (z, y) is contained in the track wall. The angle between the tangent t on the wall at point A and the z -axis is the local development angle δ . Although further etching will progress in all directions with the same rate V_b , including the surroundings of point E , it can be characterized by two processes for easier treatment, namely, the displacement for the distance d of the wall from position 1 to position 2, and formation of a sphere around point E with same diameter d . Point A moves normally onto the tangent t for a distance d to point A' with coordinates (z', y') . The track wall consists of two parts, i.e., semi-conical and spherical. These two parts intersect at points M_1 and M_2 (if the problem is considered in two dimensions). In the three-dimensional representation, intersection occurs at a circle, which is schematically shown in Fig. 2 as well as in Fig. 5.

From geometrical consideration, the relationships between the (z, y) and (z', y') coordinates are given by

$$z' = z + d \sin \delta \quad \text{and} \quad y' = y + d \cos \delta. \quad (10)$$

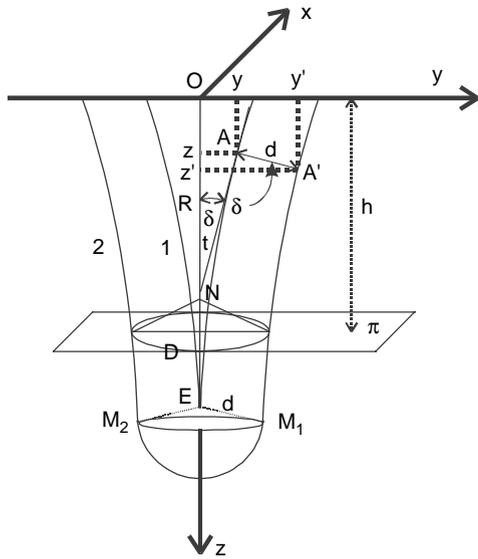


Fig. 2. An over-etched track. O and E are entrance and end points of a particle; $R = OE$ is the particle range. Line 1: wall position at the moment when the etching solution reaches point E ; d : the removed layer during over-etching; Line 2: the new wall position; M_1 and M_2 : intersection points; t : tangent on the wall at point A and δ is the local developing angle of points A and A' .

By using Eq. (7d) with $L = R$, and combining with Eq. (10), one can obtain the equation for the semi-conical part (in the over-etched phase) in two dimensions as

$$y' = F(z, R) + d \cos \delta \quad (11)$$

or

$$y' = F(z' - d \sin \delta, R) + d \cos \delta. \quad (12)$$

Now, we can formally replace y' with y , and z' with z , to get the equation for the track wall in two dimensions as

$$y = F(z - d \sin \delta, R) + d \cos \delta. \quad (13)$$

This is the equation of the track wall in two dimensions at the moment when the etchant reaches the end point of the particle trajectory.

The equation of the track wall in three dimensions is given analogously to Eq. (8) as

$$\sqrt{x^2 + y^2} = F(z - d \sin \delta, R) + d \cos \delta. \quad (14)$$

The detector surface after etching is represented by the plane π in Fig. 2. The plane π is normal to the z -axis which represents the particle trajectory. The thickness of the removed layer is denoted by h (see Fig. 2). If $z = h$ is substituted into Eq. (14), the equation of a circle in the plane π is obtained as

$$\sqrt{x^2 + y^2} = F(h - d \sin \delta, R) + d \cos \delta = D', \quad (15)$$

where D' is radius of the circle and (x, y) are the coordinates of points on the circle. The angle δ in Eqs. (14) and (15) is the local developing angle at point N (Fig. 2). All points

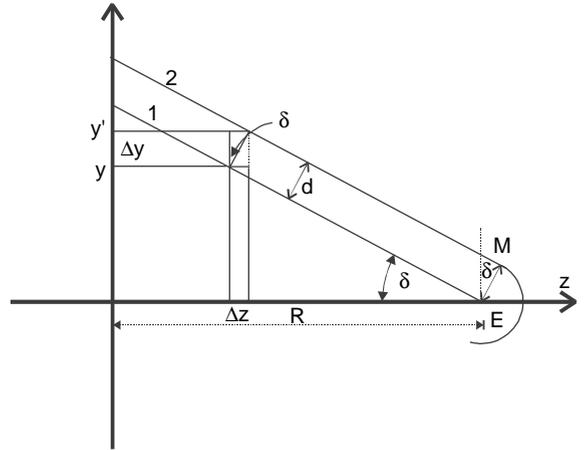


Fig. 3. Determination of the intersection point when $V = \text{constant}$.

of the track-opening contour were developed from the same point N on the particle path under the same developing angle δ . This remark can be generalized to all points with the same z and same developing angle δ . The diameter, $D = 2D'$, of the circular track opening is found as

$$D = 2D' = 2[F(h - d \sin \delta, R)] + d \cos \delta. \quad (16)$$

3.1. Special case: $V = \text{constant}$ and normal incidence

This case is depicted in Fig. 3, where the track wall in two dimensions is a line. Line 1 in Fig. 3 represents the track wall when the etchant reaches the end point E of the particle trajectory. The equation of line 1 is given as

$$y_1 = (R - z) \tan \delta, \quad (17)$$

where R is the particle range. Line 2 represents the track wall in two dimensions after prolonged etching for the removed layer d , with the equation given as

$$y_2 = (R - z + d \sin \delta) \tan \delta + d \cos \delta. \quad (18)$$

Now, Eq. (13) is transformed in the linear case through Eq. (7e) as $F(z, R) = (R - z)/(V^2 - 1)^{1/2}$. Then $F(h - d \sin \delta, R) = (R - h + d \sin \delta)/(V^2 - 1)^{1/2}$ gives the same equation as Eq. (18) with a substitution by h , i.e. $z \rightarrow h$. If $V = \text{constant}$, Eq. (16) can be transformed as

$$\frac{D}{2} = \frac{1}{\sqrt{V^2 - 1}} (R - h + d \sin \delta) + d \cos \delta. \quad (19)$$

By substituting $\sin \delta = 1/V$ and $\cos \delta = (V^2 - 1)^{1/2}/V$, one can obtain

$$\begin{aligned} \frac{D}{2} &= \frac{1}{\sqrt{V^2 - 1}} \left(R - h + \frac{d}{V} \right) + \frac{d}{V} \sqrt{V^2 - 1} \\ &= \frac{R - h + dV}{\sqrt{V^2 - 1}} \end{aligned} \quad (20)$$

or

$$D = 2 \frac{V_t t_1 - V_b(t_1 + t_2) + V_b t_2 \frac{V_t}{V_b}}{\frac{\sqrt{V_t^2 - V_b^2}}{V_b}} = 2V_b T \frac{\sqrt{V_t - V_b}}{\sqrt{V_t + V_b}} \quad (21)$$

which is the same equation for the track diameter as given by [Durrani and Bull \(1987, p. 54\)](#). In the previous equation, t_1 is the time when etching reaches point E , t_2 is over-etching time and $T = t_1 + t_2$ is the total etching time. Eqs. (8) and (14) give good results for the simplest case (normal incidence and constant V).

3.2. Intersection points M_1 and M_2

The z coordinates of intersection points can be determined in two dimensions because the problem is axially symmetric. The equation for the circle with center at point $E(0, R)$ and radius d (see Fig. 2) is

$$(z - R)^2 + y^2 = d^2. \quad (22)$$

The intersection points belong to both semi-conical and spherical parts of the track wall. Therefore, the coordinates (y_m, z_m) of the intersection points should satisfy both Eqs. (13) and (22). The coordinates of the intersection points M_1 and M_2 can be found by combining Eqs. (13) and (22) and the following was found:

$$\sqrt{d^2 - (z_m - R)^2} = F(z_m - d \sin \delta, R) + d \cos \delta. \quad (23)$$

The last equation gives the coordinate z_m of the intersection points. The radius of the intersection circle in three dimensions is given by $y_m = [d^2 - (z_m - R)^2]^{1/2}$. An analytical solution of Eq. (23) can be obtained for the simplest case where $V = \text{constant}$. One should also be able to find the common points of line 2 and a circle around E with radius d . From Fig. 3, the coordinates of the common point M are

$$z_m = R + d \sin \delta \quad \text{and} \quad y_m = d \cos \delta. \quad (24)$$

In the case of a varying function V , Eq. (23) should be used to determine the points where the semi-conical and circular parts of the wall are joined. The unknown variable z appeared on both sides of Eq. (23) and numerical iteration is needed to solve the equation.

4. Oblique incidence

In this part, the case of oblique incidence is considered. Apparently, this situation is more complicated than the previous one. However, if two transformations of the coordinate system are applied, the problem will be simplified significantly.

4.1. Semi-elliptical opening

Semi-elliptical openings are found when the detector surface after etching did not cross the part of the sphere

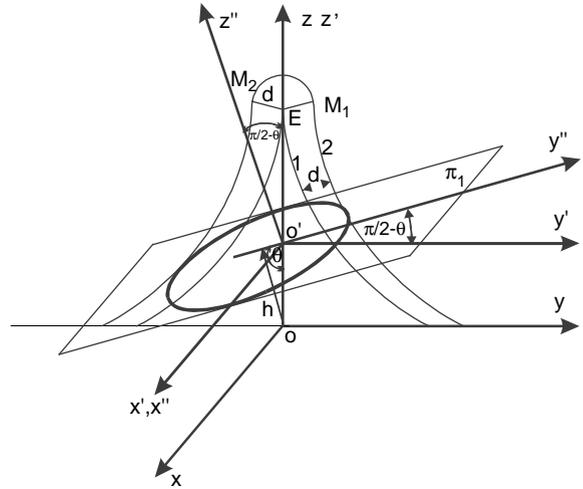


Fig. 4. Track opening in the semi-elliptical phase.

formed around point E . In this section, such kind of tracks will be considered. The geometry used for considering a semi-elliptical track opening is presented in Fig. 4. The nomenclature is the same as those in previous figures. A new parameter is the angle θ which is the incident angle with respect to the detector surface. Around the end point E of the particle trajectory, a sphere with radius d is formed which is joined with the semi-conical part of the track wall. The plane π_1 represents the detector surface after etching, and h is the thickness of the removed layer. The track is “cut” by the plane π_1 under the angle θ with respect to the particle direction (z -axis). The first step is a translation of the coordinate system (x, y, z) from point O to point O' with coordinates $O'(0, 0, z_0)$ where $z_0 = h/\sin \theta$. The newly obtained system (x', y', z') is related to the original one through the equations

$$x' = x, \quad y' = y \quad \text{and} \quad z' = z - z_0 \quad (25)$$

and

$$z = z' + z_0.$$

Eq. (14) for the track wall in the new coordinate system becomes

$$\sqrt{x'^2 + y'^2} = F(z' + z_0 - d \sin \delta, R) + d \cos \delta. \quad (26)$$

The second step is a rotation of the (x', y', z') coordinate system through an angle $(\pi/2 - \theta)$ around the x' -axis. The newly formed coordinate system (x'', y'', z'') system is related to the (x', y', z') system through the equations

$$y' = y'' \sin \theta - z'' \cos \theta \quad \text{and} \quad z' = y'' \cos \theta + z'' \sin \theta. \quad (27)$$

Eq. (26) for the track wall in the (x'', y'', z'') system is now

$$\begin{aligned} &\sqrt{x''^2 + (y'' \sin \theta - z'' \cos \theta)^2} \\ &= F(z'' \sin \theta + y'' \cos \theta + z_0 - d \sin \delta, R) \\ &\quad + d \cos \delta. \end{aligned} \quad (28)$$

The surface of the detector after etching is given as $z'' = 0$. By substituting $z'' = 0$ into Eq. (28), the intersection between the track wall and the new detector surface described by $z'' = 0$ is given as

$$\begin{aligned} &\sqrt{x''^2 + y''^2 \sin^2 \theta} \\ &= F(y'' \cos \theta + z_0 - d \sin \delta, R) + d \cos \delta. \end{aligned} \quad (29)$$

Here, x'' and y'' are the coordinate axes along the plane π_1 (both belonging to the plane π_1) and z'' is normal to π_1 . In this case, y'' is extended along the major axis of the track and x'' is normal to it. This is the equation for the contour line of the track opening in the semi-elliptical phase, where the track is rounded but has not yet passed the spherical shape. The angle δ , which appears in the Eq. (29) implicitly and varies along the contour line, makes calculations difficult. However, calculation of the contour line is facilitated by the fact that all points with the same value of z have the same developing angle δ (as emphasized before).

4.1.1. Special case

The special case in which $V = \text{constant}$ is considered again. In this case, $F(\xi, R) = (r - \xi)/(V^2 - 1)^{0.5}$. Here, Eq. (29) for the contour line becomes

$$\begin{aligned} \sqrt{x''^2 + y''^2 \sin^2 \theta} &= \frac{1}{\sqrt{V^2 - 1}} \left(R - \frac{h}{\sin \theta} - y \cos \theta + \frac{d}{V} \right) \\ &\quad + \frac{d}{V} \sqrt{V^2 - 1}, \end{aligned} \quad (30)$$

where the substitutions $z_0 = h/\sin \theta$, $\sin \delta = 1/V$ and $\cos \delta = (V^2 - 1)^{1/2}/V$ have been carried out. Further transformations have brought the following:

$$\begin{aligned} &\sqrt{x''^2 + y''^2 \sin^2 \theta} \\ &= \frac{1}{\sqrt{V^2 - 1}} \left(R - \frac{h}{\sin \theta} - y \cos \theta + dV \right). \end{aligned} \quad (31)$$

The term $R + dV$ on the right side of this equation is $R + dV = V_1 t_1 + V_b t_2 V_1/V_b = V_1(t_1 + t_2) = V_1 T = V_1 h/V_b = Vh$. After some algebraic transformations, we obtain

$$\frac{x''^2}{h^2 \frac{V \sin \theta - 1}{V \sin \theta + 1}} + \frac{(y'' + \frac{h}{\text{tg } \theta(V \sin \theta + 1)})^2}{h^2 \frac{V^2 - 1}{V \sin \theta - 1}} = 1. \quad (32)$$

This equation is that of an ellipse from which we can determine the major and minor axes as well as the shift of the ellipse center along the y'' -axis. In the special case of constant V , Eq. (29) comes up to the form of an ellipse

equation with well-known expressions for minor and major axes (see Durrani and Bull, 1987, pp. 59 and 63).

4.2. Discussion of Eq. (29)

(a) Track length (major axis)

The track length can be found from Eq. (29) when $x'' = 0$. Here, the coordinates y_1 and y_2 where the contour line crosses the y'' -axis are found as

$$y_{1,2} \sin \theta = [F(y_{1,2} \cos \theta + z_0 - d \sin \delta, R) + d \cos \delta]. \quad (33)$$

Note that unknown variables y_1 and y_2 are on both sides of Eq. (33) and iterations are needed to solve the equation. The length D of the track opening is then equal to

$$D = |y_1| + |y_2|. \quad (34)$$

(b) Track width (minor axis)

The track width cannot be found by taking $y'' = 0$ because the center of the opening is shifted along the y'' -axis. In this case the maximum of the function given in Eq. (29) should be determined by locating

$$\left(\frac{dx''}{dy''} \right)_{y_{\max}} = 0, \quad (35)$$

where y_{\max} is the value of y'' when x'' has a maximum. Then y_{\max} should be substituted into Eq. (29) to find the maximum value x_{\max} . The track width (minor axis of the track opening) is given by $d = 2x_{\max}$. Such procedures are rather complicated and impractical because the angle δ also depends on the coordinate y . A better approach is to perform calculations of x'' from Eq. (29) and to determine the maximal value of x'' by systematically changing values of y'' from y_1 to y_2 .

(c) Surface area of track opening

The surface area S of the track opening can also be found from Eq. (29) by performing the integration

$$S = 2 \int_{y_1}^{y_2} \sqrt{F^2(y \cos \theta + z_0 - d \sin \theta + d \cos \delta) - y^2 \sin^2 \theta} dy, \quad (36)$$

where y_1 and y_2 are determined by Eq. (33). Numerical integration is needed to determine S .

4.3. Track opening in transitional phase

The plane representing the detector surface after etching intersects part of the sphere formed around point E . Consequently, the track opening consists of two parts, namely, semi-elliptical and circular. The geometry, although similar to the previous case, is presented separately in Fig. 5. The two parts of the track wall, semi-conical and spherical, are joined to the circle at the points M_1 and M_2 . The plane π_2 corresponding to the detector surface will cut the track after etching, so both the semi-conical and the spherical parts will be crossed. As a result, the complicated curve, namely, circle+semi-ellipse, is formed in the π_2 plane, which is also

calculations of the surface area of track openings are enabled in all phases of track development.

The minor axis of the track opening is calculated as $d = (y_1 y_2)^{1/2}$ by Somogyi and Szalay (1973), where y_1 and y_2 are coordinates of the end points of the track openings along the y (longest) axis (same as those in Eq. (33) in this paper). This expression for the minor axis is derived based on the characteristics of an ellipse. As observed from Eq. (29), the opening is not elliptical (the term semi-elliptical was employed because of no better alternatives). The deviation from an ellipse is more serious if the angle θ is smaller. In this sense, the three-dimensional approach offers a better determination of the minor axis of the track opening.

Acknowledgements

The present research is supported by the CERG grants CityU1081/01P and CityU1206/02P from the Research Grant Council of Hong Kong.

References

- Ditlov, V., 1995. Calculated tracks in plastics and crystals. *Radiat. Meas.* 25 (1/4), 89–94.
- Durrani, S.A., Bull, R.K., 1987. *Solid State Nuclear Track Detection: Principles, Methods and Applications*. Pergamon Press, Oxford.
- Fromm, M., Chambaudet, A., Membrey, F., 1988. Data bank for alpha particle tracks in CR39 with energies ranging from 0.5 to 5 MeV recording for various incident angles. *Nucl. Tracks Radiat. Meas.* 15, 115–118.
- Hatzialekou, U., Henshaw, D.L., Fewes, A.P., 1988. Automated image analysis of alpha-particle autoradiographs of human bone. *Nucl. Instr. Methods A* 263, 504–514.
- Henke, P.R., Benton, E., 1971. On geometry of tracks in dielectric nuclear track detector. *Nucl. Instr. Methods* 97, 483–489.
- Meyer, P., Fromm, M., Chambaudet, A., Laugier, J., Makovicka, L., 1995. A computer simulation of n, p conversion and resulting proton tracks etched in CR39 SSNTD. *Radiat. Meas.* 25 (1/4), 449–452.
- Nikezic, D., 2000. Three dimensional analytical determination of the track parameters. *Radiat. Meas.* 32, 277–282.
- Nikezic, D., Kostic, D., 1997. Simulation of the track growth and determination the track parameters. *Radiat. Meas.* 28, 185–190.
- Paretzke, G.H., Benton, E., Henke, P.R., 1973. On the particle track evolution in dielectric track detectors and charge identification through track radius measurements. *Nucl. Instr. Methods* 108, 73–80.
- Somogyi, G., 1980. Development of etched nuclear tracks. *Nucl. Instr. Methods* 173, 21–42.
- Somogyi, G., Szalay, A.S., 1973. Track diameter kinetics in dielectric track detector. *Nucl. Instr. Methods* 109, 211–232.